Andreas Karwath
(original slides by Peter Flach)
Logik für Informatiker: PROLOG
Part 7: Search

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&

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Search
arc(1,2).
arc(1,8).
arc(1,6).
arc(2,7).
ar(2,12).
arc(2,4).
ar(12,9).
ar(12,15).
ar(6,3).
ar(6,11).
ar(11,0).
ar(11,5).

An example graph (e.g. a tree)
% search(Agenda,Goal) <- Goal is a goal node, and a descendant of one of the nodes on the Agenda

call(Agenda,Goal):-
next(Agenda,Goal,Rest),
goal(Goal).
call(Agenda,Goal):-
next(Agenda,Current,Rest),
children(Current,Children),
add(Children,Rest,NewAgenda),
call(NewAgenda,Goal).
Depth-first vs. breadth-first search

```prolog
search_df([Goal|Rest],Goal):-
goal(Goal).
search_df([Current|Rest],Goal):-
  children(Current,Children),
  append(Children,Rest,NewAgenda),
  search_df(NewAgenda,Goal).

search_bf([Goal|Rest],Goal):-
goal(Goal).
search_bf([Current|Rest],Goal):-
  children(Current,Children),
  append(Rest,Children,NewAgenda),
  search_bf(NewAgenda,Goal).

children(Node,Children):-
  findall(C,arc(Node,C),Children).
```
BFS vs DFS
Breadth-first search

- agenda = queue (first-in first-out)
- complete: guaranteed to find all solutions
- first solution founds along shortest path
- requires $O(B^n)$ memory

Depth-first search

- agenda = stack (last-in first-out)
- incomplete: may get trapped in infinite branch
- no shortest-path property
- requires $O(B \times n)$ memory
Loop detection I
% depth-first search with loop detection

search_df_loop([Goal|Rest], Visited, Goal):-
    goal(Goal).

search_df_loop([Current|Rest], Visited, Goal):-
    children(Current, Children),
    add_df(Children, Rest, Visited, NewAgenda),
    search_df_loop(NewAgenda, [Current|Visited], Goal).

add_df([], Agenda, Visited, Agenda).
add_df([Child|Rest], OldAgenda, Visited, [Child|NewAgenda]):-
    not(member(Child, OldAgenda)),
    not(member(Child, Visited)),
    add_df(Rest, OldAgenda, Visited, NewAgenda).
add_df([Child|Rest], OldAgenda, Visited, NewAgenda):-
    member(Child, OldAgenda),
    add_df(Rest, OldAgenda, Visited, NewAgenda).
add_df([Child|Rest], OldAgenda, Visited, NewAgenda):-
    member(Child, Visited),
    add_df(Rest, OldAgenda, Visited, NewAgenda).
% depth-first search by means of backtracking
search_bt(Goal, Goal):-
goal(Goal).
search_bt(Current, Goal):-
arc(Current, Child),
search_bt(Child, Goal).

% backtracking depth-first search with depth bound
search_d(D, Goal, Goal):-
goal(Goal).
search_d(D, Current, Goal):-
D>0, D1 is D-1,
arc(Current, Child),
search_d(D1, Child, Goal).

Backtracking search
Iterative deepening

search_id(First, Goal):-
    search_id(1, First, Goal). % start with
    % depth 1

search_id(D, Current, Goal):-
    search_d(D, Current, Goal).

search_id(D, Current, Goal):-
    D1 is D+1, % increase depth
    search_id(D1, Current, Goal).

- combines advantages of breadth-first search (complete, shortest path) with those of depth-first search (memory-efficient)
• Problem representation: the vertices of the graph represent the states of the problem, the edges represent the transitions, leading from one state to the next.

• Problem solving: finding a path from the initial state to the final state by application of a sequence of transition rules

• Goal:
  general framework for the solution of such problems using depth-first search. Formulated generally, such that arbitrary problems within the framework can be solved
% solve_dfs(State,Visited,Transitions) :-
% Transitions is the sequence of transitions
% to reach a desired final state from the
% current State. Visited contains the states
% visited previously.

solve_dfs(State,Visited,[[]]) :-
    final_state(State).

solve_dfs(State,Visited,[[Transition|Transitions]]) :-

    transition(State,Transition),
    update(State,Transition,State1),
    legal(State1),
    not(member(State1,Visited)),
    solve_dfs(State1,[[State1|Visited]],Transitions).

% Testing the framework:
test_dfs(Problem,Transitions) :-
    initial_state(Problem,State),
    solve_dfs(State,[[State]],Transitions).
transition(State, Transition): binary predicate specifying the states; Transition is applicable to State

update(State, Transition, State1): applying Transition to State leads to State1

legal(State): State is a legal state

not member(State, Visited): cycles can be detected; all visited states are stored in Visited

solve_dfs(State, Visited, Transitions): incrementally the sequence of transitions, from the initial to the final state, is built

Application: search in a state space
Representing a problem:

- The states have to be represented.
- For the predicates `transition`, `update`, `legal`, etc. axioms have to be found.

Application: search in a state space
Example: the water jugs problem

There are two water jugs of capacity 8 and 5 liters with no markings, and the problem is to measure out exactly 4 liters from a vat containing 20 liters (or some other larger number). The possible operations are filling up a jug from the vat, emptying a jug into the vat, and transferring the contents of one jug to another until either the pouring jug is emptied completely, or the other jug is filled to capacity.
Required facts:

capacity(C, JC), for C equal j1 or j2;
jugs(C1, C2), where C1 and C2 give the current contents of the jugs

the initial state: jugs(0, 0)

the final states: jugs(4, 0) or jugs(0, 4)
Six kinds of **transitions**: Filling up and emptying a jug, and transferring the contents of one jug to the other. E.g.,

\[ \text{transition}(\text{jugs}(C1,C2),\text{fill}(j1)) \]

Transitions for emptying can be optimized (emptying already empty jugs is useless...)

**update** predicate:
- for emptying and filling: easy
- for transferring: enough capacity?

Checking for **legal** states: trivial
initial_state(jugs, jugs(0,0)).

final_state(jugs(4,C2)).
final_state(jugs(C1,4)).

transition(jugs(C1,C2), fill(j1)).
transition(jugs(C1,C2), fill(j2)).
transition(jugs(C1,C2), empty(j1)) :- C1 > 0.
transition(jugs(C1,C2), empty(j2)) :- C2 > 0.
transition(jugs(C1,C2), transfer(j2,j1)).
transition(jugs(C1,C2), transfer(j1,j2)).
update(jugs(C1,C2),empty(j1),jugs(0,C2)).
update(jugs(C1,C2),empty(j2),jugs(C1,0)).
update(jugs(C1,C2),fill(j1),jugs(Capacity,C2)) :-
    capacity(j1,Capacity).
update(jugs(C1,C2),fill(j2),jugs(C1,Capacity)) :-
    capacity(j2,Capacity).
update(jugs(C1,C2),transfer(j2,j1),jugs(W1,W2)) :-
    capacity(j1,Capacity),
    Water is C1 + C2,
    Overhang is Water - Capacity,
    adapt(Water,Overhang,W1,W2).
update(jugs(C1,C2),transfer(j1,j2),jugs(W1,W2)) :-
    capacity(j2,Capacity),
    Water is C1 + C2,
    Overhang is Water - Capacity,
    adapt(Water,Overhang,W2,W1).
adapt(Water,Overhang,Water,0) :- Overhang =< 0.
adapt(Water,Overhang,C,Overhang) :-
    Overhang > 0,
    C is Water - Overhang.

legal(jugs(C1,C2)).

capacity(j1,8).
capacity(j2,5).
?- test_dfs(jugs,T).

T = [fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2)] ;

T = [fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2), empty(j2)] ;

T = [fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2)] ;

T = [fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1), transfer(j1,j2), empty(j2), transfer(j1,j2), fill(j1)] ;

no