Principles of AI Planning

Complexity of nondeterministic planning with partial observability

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Introduction

- Earlier, we showed how deterministic Turing Machines with polynomial space can be translated to deterministic planning tasks.
- Later, we saw how alternation in Turing Machines can be translated into nondeterminism in the planning task.
- We also saw how exponential space in Turing Machines can be modeled by using unobservable planning tasks.

Now, we will combine the latter two proof techniques to show that nondeterministic planning with partial observability is 2-EXP-complete.

The strong planning problem for partial observability

PartialPlanEx (plan existence for partial observability)

Given:	nondeterministic planning task $\langle A, I, O, G, V \rangle$
Question:	Is there a strong plan for the task?

- We do not consider the analog of the bounded plan existence problem (PlanLen).

Membership in 2-EXP

PartialPlanEx $\in$ 2-EXP

For input $\mathcal{T}$:

- Use the reduction algorithm presented in the previous lecture to generate an equivalent nondeterministic planning task with full observability $\mathcal{T}'$ in exponential time.
  - This requires exponential time and creates a task of exponential size in $\|\mathcal{T}\|$.
- Solve the resulting task using an EXP algorithm.
  - This requires exponential time in $\|\mathcal{T}'\|$, which is doubly exponential in $\|\mathcal{T}\|$.

Thus, the problem can be solved within 2-EXP.

Reduction idea

- We want to prove that PARTIALPLANEX is 2-EXP-hard.
- To do this, we need to reduce all problems in 2-EXP to PARTIALPLANEX.
- A problem is in 2-EXP iff there exists a DTM that accepts instances of the problem in doubly exponential time.
- Equivalently, by Chandra et al.'s theorem, a problem is in 2-EXP iff there exists an ATM that accepts instances of the problem in exponential space (since AEXPSPACE $=$ 2-EXP).
- We exploit the latter relationship by providing a generic reduction from word acceptance for ATMs with exponential space to PARTIALPLANEX.

- For a fixed polynomial p, given ATM M and input w, generate planning task which is solvable by a strong plan iff M accepts w in space $2^{p(|w|)}$.

For simplicity, we only consider ATMs with two restrictions (no loss of generality):

- ATM never moves to the left of the initial head position.
- If several ATM transitions are possible in universal state q reading the symbol a, then the resulting state q' is different for all these transitions.

(The second restriction is so that the planning agent can know which transition was taken by looking at the current state.)

Idea of the reduction
Dealing with alternation

- Existential states of the ATM are modeled by states of the planning task where there are several applicable operators to choose from.
- Universal states of the ATM are modeled by states of the planning task where there is a single applicable operator with a nondeterministic effect.

Idea of the reduction

Dealing with exponential space

- Only keep track of the contents of one tape cell $\rightsquigarrow$ watched tape cell.
- Which tape cell is watched is unobservable.
- $\rightsquigarrow$ Plan must work correctly for all possible choices.
- $\rightsquigarrow$ Plan must remain faithful to the TM computation.

Reduction: state variables

Let p be a polynomial such that $2^{p(n)} - 2$ is a space bound for inputs of size n.

Given: ATM $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ and input $w_1 \dots w_n$.

State variables

Convention:
Use $\overline{\text{bars}}$ to denote vectors of $p(n)$ state variables encoding a number in the range $0 \dots, 2^{p(n)} - 1$.

- $\overline{\text{state}_q}$ for all $q \in Q$ – current TM state
- $\overline{\text{head}}$ – head position
- $\overline{\text{watched}}$ – position of the watched tape cell
- content_a for all $a \in \Sigma_\Box$ – contents of the watched tape cell

The $\overline{\text{watched}}$ variables are unobservable.
All other variables are observable.

Spelling it out

- $(\overline{\mathsf{head}} = 1) \equiv \neg\mathsf{head}_1 \land \cdots \land \neg\mathsf{head}_{p(n)-1} \land \mathsf{head}_{p(n)}$
- $(\overline{\mathsf{head}} = 5) \equiv \neg\mathsf{head}_1 \land \cdots \land \neg\mathsf{head}_{p(n)-3}$
  $\land\, \mathsf{head}_{p(n)-2} \land \neg\mathsf{head}_{p(n)-1} \land \mathsf{head}_{p(n)}$
- $(\overline{\mathsf{head}} = \overline{\mathsf{watched}}) \equiv$
  $(\neg\mathsf{head}_1 \lor \mathsf{watched}_1) \land (\mathsf{head}_1 \lor \neg\mathsf{watched}_1)$
  $\land\, (\neg\mathsf{head}_2 \lor \mathsf{watched}_2) \land (\mathsf{head}_2 \lor \neg\mathsf{watched}_2)$
  $\land \ldots$
- $\overline{\mathsf{head}} := \overline{\mathsf{head}} + 1 \equiv$
  $(\neg\mathsf{head}_{p(n)} \rhd \mathsf{head}_{p(n)})$
  $\land\, ((\neg\mathsf{head}_{p(n)-1} \land \mathsf{head}_{p(n)}) \rhd (\mathsf{head}_{p(n)-1} \land \neg\mathsf{head}_{p(n)}))$
  $\land \ldots$
- $\overline{\mathsf{head}} := \overline{\mathsf{head}} - 1 \equiv \ldots$

Reduction: initial state formula

Initial state formula

$$
\begin{aligned}
I = {} & \mathsf{state}_{q_0} \wedge \bigwedge_{q \in Q \setminus \{q_0\}} \neg \mathsf{state}_q \\
& \wedge \overline{\mathsf{head}} = 1 \\
& \wedge \left(\bigwedge_{i=1}^{n} ((\overline{\mathsf{watched}} = i) \rightarrow \mathsf{content}_{w_i}) \right) \\
& \wedge (\overline{\mathsf{watched}} = 0 \vee \overline{\mathsf{watched}} > n) \rightarrow \mathsf{content}_{\Box} \\
& \wedge \bigwedge_{a \in \Sigma_{\Box}} \bigwedge_{a' \in \Sigma_{\Box} \setminus \{a\}} \neg (\mathsf{content}_a \wedge \mathsf{content}_{a'})
\end{aligned}
$$

Note: watched tape cell unspecified

Reduction: operators

Operators

For each transition rule $((q, a), (q', a', \Delta)) \in \delta$, define:

- precondition:
  $$\begin{aligned}\mathsf{pre}_{q,a} := \mathsf{state}_q &\\ &\wedge ((\overline{\mathsf{head}} = \overline{\mathsf{watched}}) \rightarrow \mathsf{content}_a)\\ &\wedge \overline{\mathsf{head}} > 0\\ &\wedge \overline{\mathsf{head}} < 2^{p(n)} - 1\end{aligned}$$
- effect:
  $$\begin{aligned}\mathsf{eff}_{q,a,q',a',\Delta} := \neg\mathsf{state}_q \wedge \mathsf{state}_{q'} &\\ &\wedge ((\overline{\mathsf{head}} = \overline{\mathsf{watched}}) \rhd \neg\mathsf{content}_a)\\ &\wedge ((\overline{\mathsf{head}} = \overline{\mathsf{watched}}) \rhd \mathsf{content}_{a'})\\ &\wedge (\overline{\mathsf{head}} := \overline{\mathsf{head}} + \Delta)\end{aligned}$$
  If $q = q'$, omit the effects in the first line.
  If $a = a'$, omit the effects in the second and third line.

Reduction: operators (continued)

Operators (ctd.)

For existential states $q \in Q_\exists$, $a \in \Sigma_\Box$:
Let $(q'_j, a'_j, \Delta_j)_{j \in \{1,\dots,k\}}$ be those triples with $((q, a), (q'_j, a'_j, \Delta_j)) \in \delta$.

For each $j \in \{1, \dots, k\}$, introduce one operator:

- precondition: $\mathrm{pre}_{q,a}$
- effect: $\mathrm{eff}_{q,a,q'_j,a'_j,\Delta_j}$

Reduction: operators (continued)

Operators (ctd.)

For universal states $q \in Q_\forall$, $a \in \Sigma_\Box$:
Let $(q'_j, a'_j, \Delta_j)_{j \in \{1,\dots,k\}}$ be those triples with $((q,a),(q'_j, a'_j, \Delta_j)) \in \delta$.

Introduce only one operator:

- precondition: $\mathsf{pre}_{q,a}$
- effect: $\mathsf{eff}_{q,a,q'_1,a'_1,\Delta_1} | \dots | \mathsf{eff}_{q,a,q'_k,a'_k,\Delta_k}$

Reduction: goal

Goal

$G = \bigvee_{q \in Q_Y} \mathsf{state}_q$

2-EXP-completeness of strong planning with partial observability

Theorem (Rintanen, 2002)

PARTIALPLANEX *is 2-EXP-complete.*

Proof.

Membership in 2-EXP has been shown by providing doubly exponential-time algorithms that generate strong plans (and decide if one exists as a side effect).

Hardness follows from the previous generic reduction for ATMs with exponential space bound and Chandra et al.'s theorem (showing AEXPSPACE = 2-EXP). □

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Summary

- Nondeterministic planning with partial observability is very hard.