

Principles of AI Planning

Nondeterministic planning with partial observability

Malte Helmert Bernhard Nebel

Albert-Ludwigs-Universität Freiburg

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Nondeterministic planning with partial observability

Planning with **partial** observability is harder than both the fully observable and unobservable cases:

- **Memoryless** plans (where the next action to take only depends on the current situation) as in the fully observable case are not sufficient.
 - Of course, we cannot define a memoryless plan based on individual states because limited observability makes some states indistinguishable.
 - It is also not sufficient to consider memoryless plans where the action to take is based on the current observation class.
- **Conformant** (i.e., non-branching) plans as in the unobservable case are also clearly not powerful enough.

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Strong planning

- We will (mostly) consider the **strong** planning problem.
- Generalizations to the strong cyclic planning are similar to the fully observable case.

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Similar to other variants of the planning problem, there are three major approaches to nondeterministic planning with partial observability:

- Reduction to another problem
- Forward search
- Backward search

We will consider one example for each of these.

Algorithms

Three approaches

Reduction to another problem:

- Reduce to planning with full observability.

Forward search (progression):

- Define the search space as an AND/OR tree.
- Define a heuristic function for such trees.
- Use a tree search algorithm such as AO* or Proof Number Search.

Backward search (regression):

- Start from the set of goal states.
- Find state sets from which already generated state sets can be reached by applying operators and making observations.

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Reduction to fully observable case

- Memoryless plans are not sufficient for the partially observable case because a plan must take into account the **knowledge** collected in previous observations etc.
- During plan execution, this knowledge is represented in the **current belief state**.
- One idea for solving a partially observable task \mathcal{T} is to map it to a fully observable task \mathcal{T}' where each **belief state of \mathcal{T}** corresponds to a **state of \mathcal{T}'** .

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Reduction to fully observable case

State variables

Let $\mathcal{T} = \langle A, I, O, G, V \rangle$ be the input task with state set S .
We define the fully observable task $\mathcal{T}' = \langle A', I', O', G', A' \rangle$.

State variables

- For each state $s \in S$, there is one state variable $v_s \in A'$.
- Intuition: v_s is true in a state of \mathcal{T}' iff it is **possible** that we are currently in s .
- Formally: $A' := \{ v_s \mid s \in S \}$

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Initial state formula

Let $\mathcal{T} = \langle A, I, O, G, V \rangle$ be the input task with state set S .
We define the fully observable task $\mathcal{T}' = \langle A', I', O', G', A' \rangle$.

Initial state formula

- The initial state of \mathcal{T}' is fully deterministic (in terms of A'), as there is only one possible initial belief state in \mathcal{T} .
- For all states s in the initial belief state of \mathcal{T} , variable v_s is initially true. Other variables are initially false.
- Formally: $I' := \bigwedge_{s \in S, s \models I} v_s \wedge \bigwedge_{s \in S, s \not\models I} \neg v_s$.

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Goal formula

- A goal belief state of \mathcal{T} is one where **all possible states** satisfy G .
- This is equivalent to saying that no state in the current belief state violates G .
- We can express that by saying that none of the variables v_s for states s violating G are true.
- Formally: $G' := \bigwedge_{s \in S, s \not\models G} \neg v_s$.

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Operators (preconditions)

- Each operator $o = \langle c, e \rangle \in O$ is translated to an operator $o' = \langle c', e' \rangle \in O'$.
- To test whether operator o is applicable, we must verify that all states in the current belief state satisfy c .
- Again, this is equivalent to saying that no state in the current belief state violates c .
- Formally: $c' := \bigwedge_{s \in S, s \not\models c} \neg v_s$.

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Operators (effects)

- Each operator $o = \langle c, e \rangle \in O$ is translated to an operator $o' = \langle c', e' \rangle \in O'$.
- After applying operator o , we can possibly be in state $s \in S$ iff we were previously in some state in which o is applicable and from which applying o can lead to s .
- This is modeled by an effect
$$((\bigvee_{t \in \text{preimg}_o(s)} v_t) \triangleright v_s) \wedge (\neg(\bigvee_{t \in \text{preimg}_o(s)} v_t) \triangleright \neg v_s).$$
- Formally:
$$e' := \bigwedge_{s \in S} (((\bigvee_{t \in \text{preimg}_o(s)} v_t) \triangleright v_s) \wedge (\neg(\bigvee_{t \in \text{preimg}_o(s)} v_t) \triangleright \neg v_s)).$$

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Reduction to fully observable case

Done?

- We have translated state variables, initial state formula, goal formula and operators.

Is that it?

- So far, our translation is **independent** of the set of observable variables V !
- Moreover, the resulting planning task is **deterministic**!

Is there an error in our modeling?

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Reduction to fully observable case

Not done

Is there an error in our modeling?

- No, but it is not **complete** yet: There are solvable partially observable tasks \mathcal{T} for which \mathcal{T}' (as defined so far) is unsolvable.
- The reason for this is that he have not yet modeled the possibility of **observing** state variables.

Modeling observations requires introducing **nondeterminism** in \mathcal{T}' .

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Observations

Let $\mathcal{T} = \langle A, I, O, G, V \rangle$ be the input task with state set S .
We define the fully observable task $\mathcal{T}' = \langle A', I', O', G', A' \rangle$.

Observations

- In general, our formalism allows observations to be general formulas over V . However, it is sufficient to only consider **atomic observations** $u \in V$.
- If we observe u in a belief state b , we can end up in two different belief states: one containing exactly the states of b where u is true, and one containing exactly the states of b where u is false.
- In other words, either the belief states where u is false or the belief states where u is true are **ruled out**.
- Formally: Translate observation of $u \in V$ into an operator $\langle \top, e'_u \rangle \in O'$ with $e'_u := (\bigwedge_{s \in S, s \not\models u} \neg v_s) \mid (\bigwedge_{s \in S, s \models u} \neg v_s)$.

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Reduction to fully observable case

Discussion

- Note that the reduction works both for strong and for strong cyclic planning.
- The reduction has a significant drawback: Since it introduces as many **state variables** as there are **states** in the original task, the resulting problem is exponentially larger than the original one.
- This will usually not be practical.
- On the other hand, there does not really exist **any** truly “practical” algorithm for nondeterministic planning with partial observability.

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Reduction to fully observable case

Complexity result

- Using an exponential-time planning algorithm for fully observable planning, \mathcal{T}' can be solved in time $O(c^{\|\mathcal{T}'\|})$, and $\|\mathcal{T}'\| = O(c^{\|\mathcal{T}\|})$.
- Thus, we have a double-exponential ($O(c^{c^{\|\mathcal{T}\|}})$) algorithm for nondeterministic planning for partial observability.
- We will later prove that this is worst-case optimal.

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Search in AND/OR trees

In forward search, plans are represented as trees whose nodes represent the situations arising during plan execution.

- The root node represents the initial situation.
- **OR nodes** correspond to choosing and applying operators.
 - Note how these relate to operators in \mathcal{T}' in the earlier reduction.
- **AND nodes** correspond to making observations.
 - Note how these relate to nondeterminism in \mathcal{T}' in the earlier reduction.

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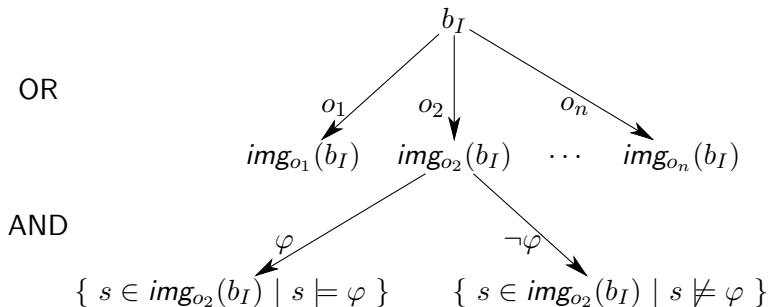
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Search in AND/OR trees

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AND/OR trees

Formal definition

Definition

An **AND/OR tree** is a labeled rooted tree where

- internal nodes are labeled with (\wedge) or (\vee) (**AND nodes/OR nodes**), and
- leaves are labeled with (\top) or (\perp) (**true leaves/false leaves**).

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AND/OR trees

Truth value

Definition

An AND/OR tree **evaluates to true** iff

- it is a true leaf,
- it is an OR node with a child that evaluates to true, or
- it is an AND node whose children all evaluate to true.

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Partial plan trees

Definition

A **partial plan tree** for a nondeterministic planning task $\langle A, I, O, G, V \rangle$ with state set S is an AND/OR tree with the following properties:

- Each node n has an associated belief state $b(n)$.
- If n is the root node, then $b(n) = \{ s \in S \mid s \models I \}$.
- A leaf node n is labeled with (\top) iff $b(n) \models G$. In this case it is called a **goal node**, otherwise an **open node**.
- ...

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Partial plan trees

Definition (ctd.)

A **partial plan tree** for a nondeterministic planning task $\langle A, I, O, G, V \rangle$ with state set S is an AND/OR tree with the following properties:

- ...
- An OR node n (also called an **operator node**) has one child n_o for each operator $o \in O$ applicable in $b(n)$, with associated belief state $b(n_o) = \text{app}_o(b(n))$.
- An AND node n (also called an **observation node**) has an associated formula $\varphi(n)$ over V . It has two children:
 - n^\top with $b(n^\top) = \{ s \in b(n) \mid s \models \varphi \}$
 - n^\perp with $b(n^\perp) = \{ s \in b(n) \mid s \not\models \varphi \}$.

Forward planning as search in partial plan trees

- Clearly, a partial plan tree represents a strategy.
- This strategy is a strong plan iff the tree evaluates to true.

We thus obtain a (nondeterministic) **forward search algorithm**:

Forward search in partial plan trees

def expand-tree(T):

Set T to the partial plan tree for \mathcal{T} that consists
of a single leaf, labeled with the initial belief state.

while T evaluates to false:

Choose some open leaf n in T .

Replace n by an operator or observation node,
adding the necessary children to T .

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Search in AND/OR trees

Issues

- There is a **conflict** between **plan size** and **observing**:
 - With many observations, plans become very big.
 - With few observations, it may be impossible to find a plan.

Trying out all possible ways to branch is not feasible.

No good general solutions to this problem exist.

- AND-OR search algorithms use heuristics for making branching decisions.
 - But they do not really work well. . .

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Backward search algorithms

- Backward search algorithms are similar in flavour to the ones for fully observable problems.
- Backward steps with **operator application**:
 - Compute strong preimages.
- Backward steps with **observations**:
 - Compute union of belief states from disjoint observational classes.
 - Note: Can always take subsets of solved belief states to make them disjoint.

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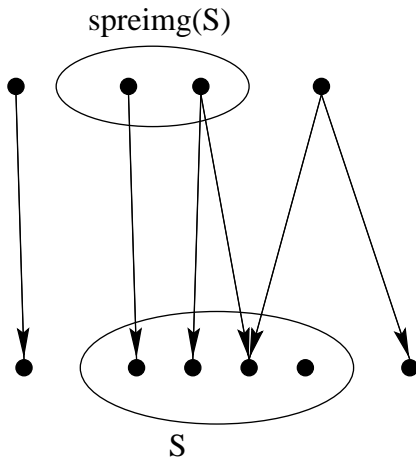
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Backward search algorithms

Regression: strong preimages



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Observations in backward search

- Let C_1, \dots, C_n be different observational classes.
- Let B_1, \dots, B_n be belief states with $B_i \subseteq C_i$ for all $i = 1, \dots, n$ for which we have a solution plan.
- Then we can find a plan for $B = B_1 \cup \dots \cup B_n$ by first observing in which class C_i we are and then applying the corresponding plan for B_i .

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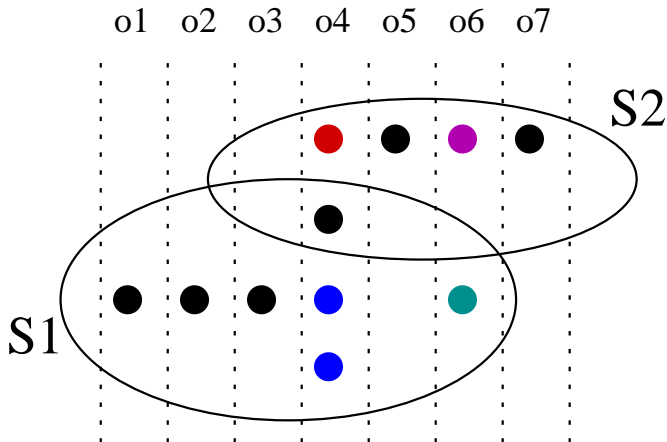
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Observations in backward search

Example: Combining two belief states



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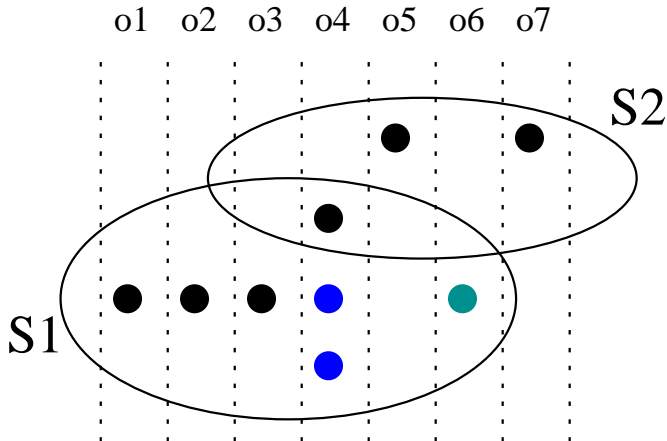
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Observations in backward search

Example: Combining two belief states, option 1



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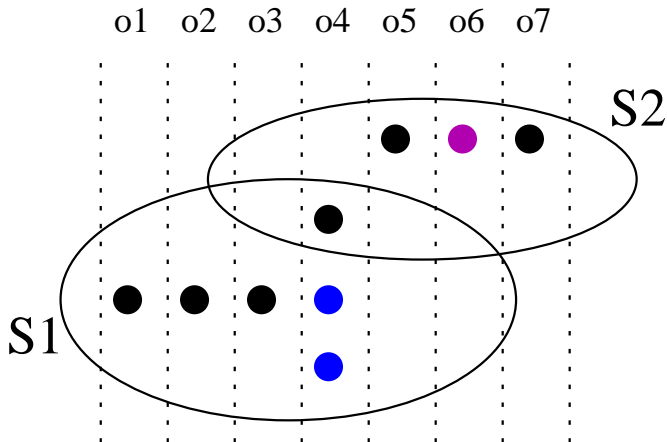
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Observations in backward search

Example: Combining two belief states, option 2



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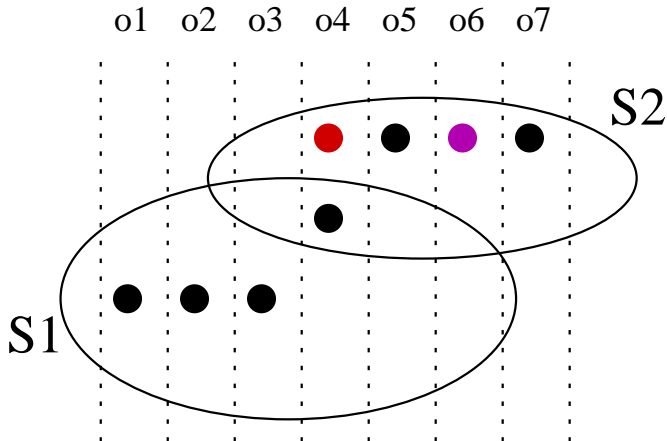
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Observations in backward search

Example: Combining two belief states, option 3



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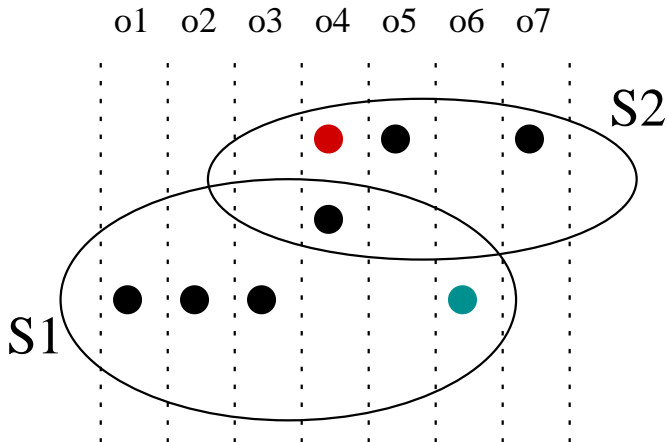
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Observations in backward search

Example: Combining two belief states, option 4



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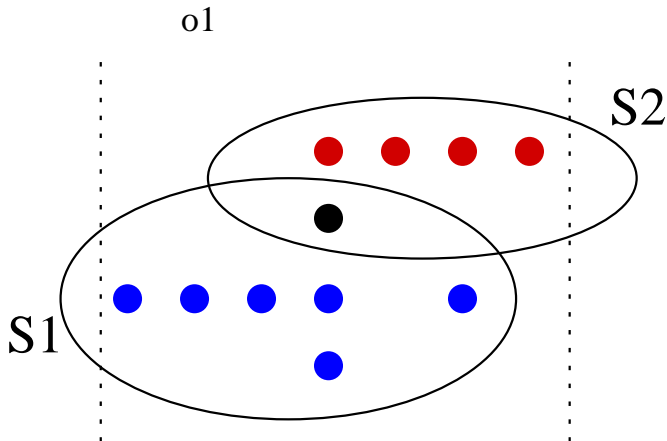
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No observability \Rightarrow no branching

Only one observational class: no choice between subplans



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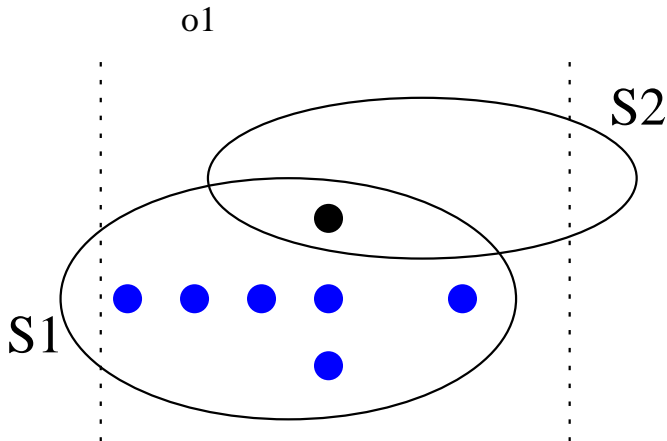
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No observability \Rightarrow no branching

No choice between subplans during execution: option 1



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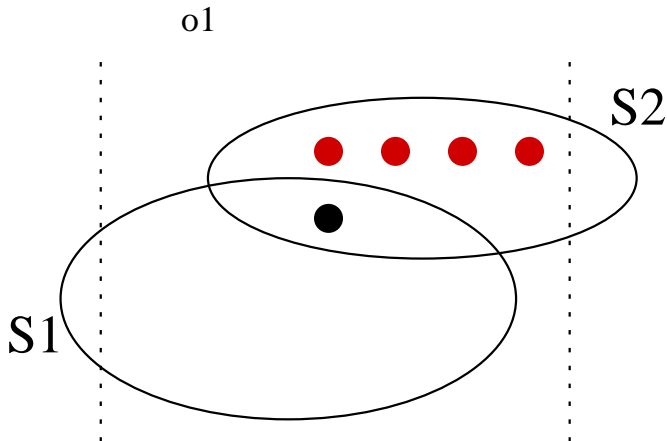
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No observability \Rightarrow no branching

No choice between subplans during execution: option 2



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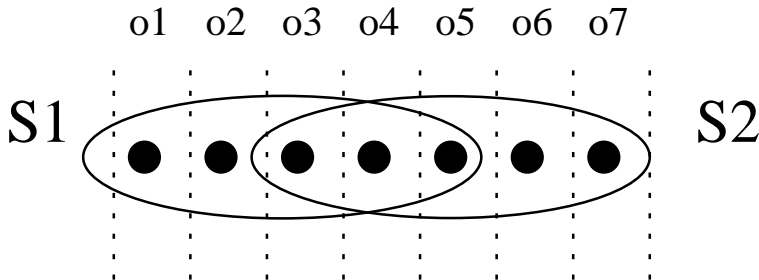
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Full observability \Rightarrow arbitrary branching

A different plan can be used for every state



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A systematic backward algorithm

Idea: always split belief states into all observational classes.

Initially, the set of solved belief states includes the set $b_G \cap C_i$ for each observational class C_i , where b_G is the belief state containing all states satisfying the goal.

Then iterate the following steps:

- 1 Pick one belief state b_i for each observational class and compute their union b .
- 2 If b includes all initial states \rightsquigarrow solution.
- 3 Otherwise, compute the strong preimage of b with respect to some operator o .
- 4 Split the resulting set of states to belief states for different observational classes and add them to the set of solved belief states.

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- Blocks world with three blocks
- Goal: all blocks are on the table
- Only the variables `clear(X)` are observable.
- A block can be moved onto the table if the block is clear.
- 8 observational classes corresponding to the 8 valuations of $\{\text{clear}(A), \text{clear}(B), \text{clear}(C)\}$ (one of the valuations does not correspond to a blocks world state).

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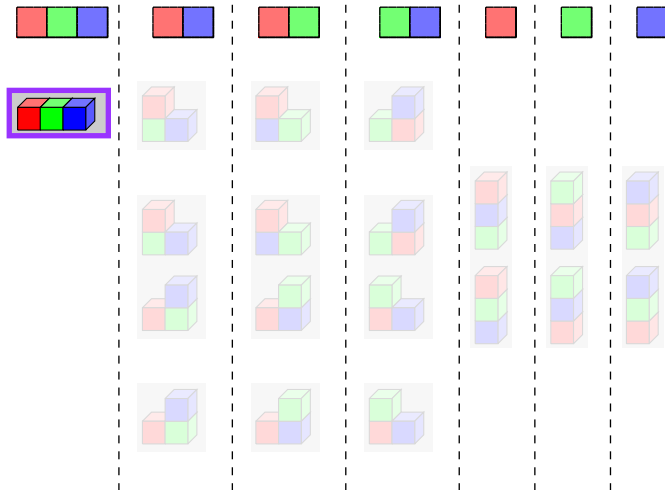
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Plan construction by backward search

Example: goal belief state



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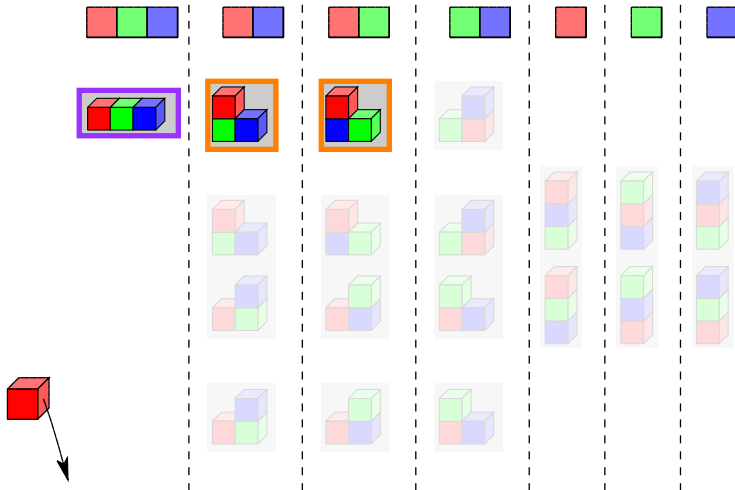
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Plan construction by backward search

Example: backward step with red-block-onto-table



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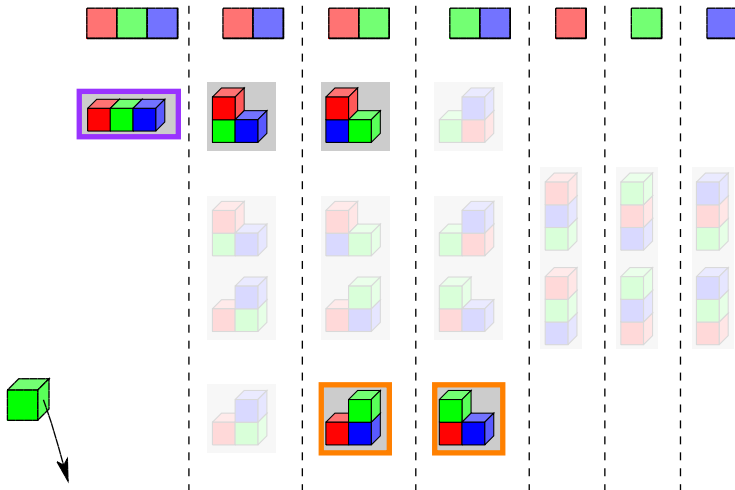
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Plan construction by backward search

Example: backward step with green-block-onto-table



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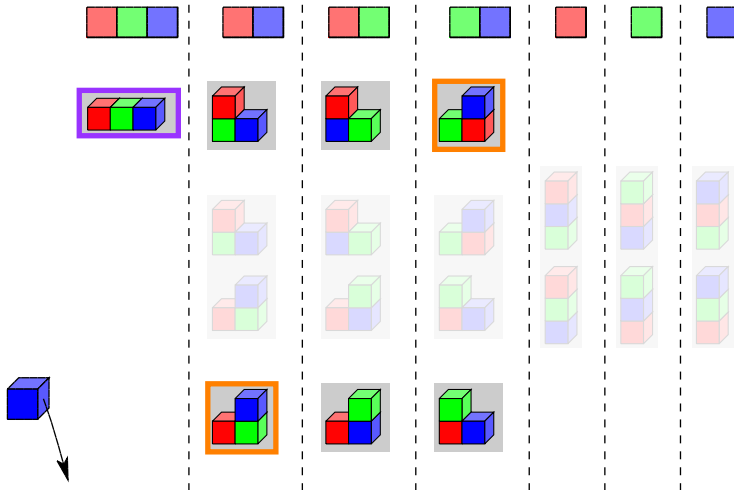
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Example: backward step with blue-block-onto-table



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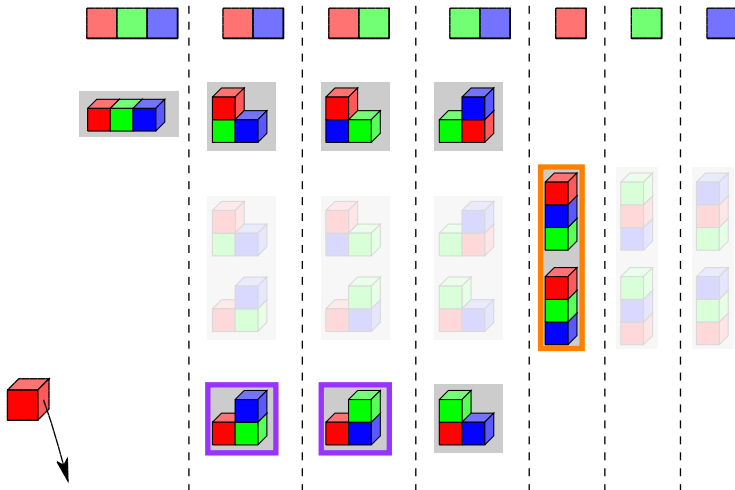
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Plan construction by backward search

Example: backward step with red-block-onto-table



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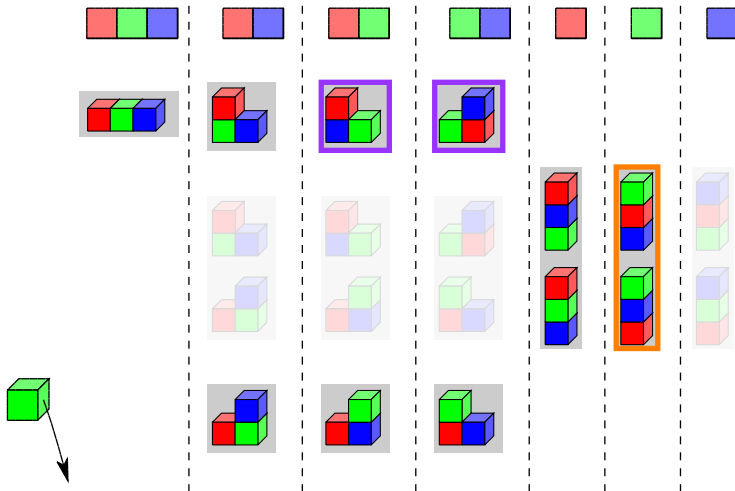
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Example: backward step with green-block-onto-table



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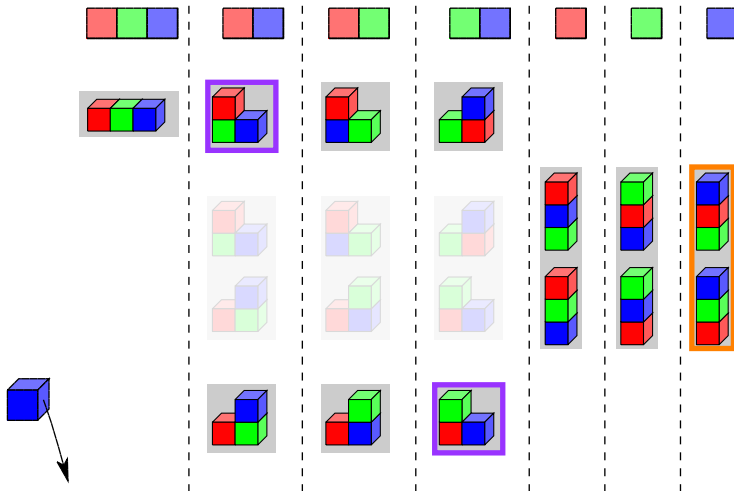
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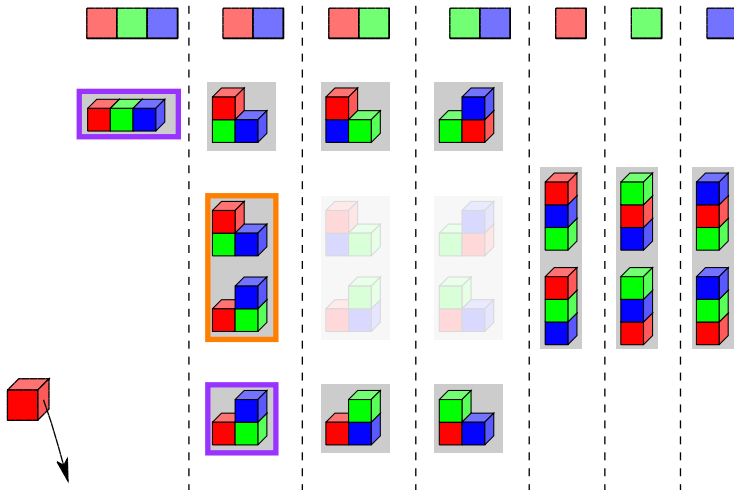
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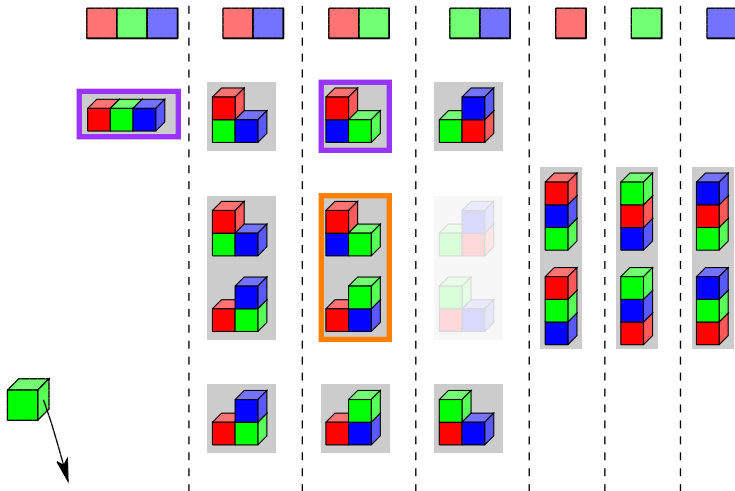
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- Planning with **partial observability** in general requires more general classes of plans than the **fully observable** and **unobservable** special cases.
- It appears to be significantly harder.
- Algorithmic ideas are similar to the simpler cases:
 - Reduction to full observability by viewing **belief states** as states.
 - Forward search in AND/OR trees.
 - Dynamic-programming style backward construction of solvable belief states, starting from goal belief states.

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