# Principles of AI Planning

February 2, 2007 — Complexity of conformant planning





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## Complexity of conformant planning


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## Motivation

- We have seen that non-determinism adds to the complexity in the case of full observability (PSPACE $\rightsquigarrow$ EXP)
- Conformant planning probably adds also to the complexity because of the larger search space
- But how much? Is it easier or harder than non-deterministic, fully observable planning?
- Again, the main motivation is to determine the limit of what is possible algorithmically: Should we try to develop a polynomial algorithm? Or would Local search algorithm suffice?





## Outline

- It turns out that conformant planning is EXPSPACE-complete
- In other words, it is (probably) more complicated than planning in the fully observable case (which is EXP-complete)
- The basic proof idea is very similar to the PSPACE-completeness proof for deterministic planning.
- The main difficulty is that we have to deal with an exponentially larger tape, which has to be fully instantiated, i.e., we need exponentially many operators

## The conformant planning problem

**ConformantPlanEx (conformant plan existence)**

GIVEN: nondeterministic planning task $\langle A, I, O, G, V \rangle$ with no observability ($V = \emptyset$)

QUESTION: Is there a conformant plan for the task?

- We do not consider the analog of the bounded plan existence problem (PlanLen).





## Membership in EXPSPACE

**ConformantPlanEx $\in$ EXPSPACE**

Generate a classical propositional planning task which has one state variable for each state of the input task.

- states of the generated planning task correspond to belief states of the input task
- operators, initial states, goal "easy" (wrt. the unfolded state space) to convert

$\rightsquigarrow$ exponential-time reduction to a problem in PSPACE
$\rightsquigarrow$ EXPSPACE algorithm





## Hardness for EXPSPACE

Idea:

- generic reduction for DTMs with exponential space
- TM states and tape head position easily representable with polynomially many state variables

Problem:

- must encode exponentially many tape cell contents with polynomially many state variables





## Hardness for EXPSPACE (continued)

The trick:

- only keep track of the contents of one tape cell $\rightsquigarrow$ watched tape cell
- which tape cell is watched is unobservable
- $\rightsquigarrow$ plan must work correctly for all possible choices
- $\rightsquigarrow$ plan must remain faithful to the TM computation

# Reduction: State Variables

Let $p$ be a polynomial such that $2^p$ is a space bound.

Given DTM $\langle \Sigma, \Box, Q, q_0, I, \delta \rangle$ and input $w_0 \dots w_n$,
define relevant tape positions $X = \{0, \dots, 2^{p(n)} - 1\}$.

## State variables

Convention:
Use $\overline{\text{bars}}$ to denote vectors of $p(n)$ state variables encoding a number in the range $0 \dots, 2^{p(n)} - 1$.

- $\text{state}_q$ for all $q \in Q$
- $\overline{\text{head}}$ – the head position
- $\text{content}_a$ for all $a \in \Sigma_\Box$
- $\overline{\text{watched}}$ – the position of the *watched* tape cell





# Spelling it out

- $\overline{\text{head}} \equiv \text{head}_1 \dots \text{head}_{p(n)}$
- $(\overline{\text{head}} = 1) \equiv \neg\text{head}_1 \wedge \dots \neg\text{head}_{p(n)-1} \wedge \text{head}_{p(n)}$
- $(\overline{\text{head}} = \overline{\text{watched}}) \equiv (\neg\text{head}_1 \vee \text{watched}_1) \wedge (\text{head}_1 \vee \neg\text{watched}_1) \wedge \dots$
- $\overline{\text{head}} := \overline{\text{head}} + 1 \equiv (\neg\text{head}_{p(n)} \rhd \text{head}_{p(n)}) \wedge$
  $(\neg\text{head}_{p(n)-1} \wedge \text{head}_{p(n)} \rhd \text{head}_{p(n)-1} \wedge \neg\text{head}_{p(n)}) \dots$
- $\overline{\text{head}} := \overline{\text{head}} - 1 \equiv \dots$





# Reduction: Initial State Formula

## Initial state formula

$$\begin{aligned}
I = {} & \text{state}_{q_0} \wedge \bigwedge_{q \in Q \setminus \{q_0\}} \neg\text{state}_q \\
& \wedge \overline{\text{head}} = 0 \\
& \wedge \bigwedge_{i=0}^{n} ((\overline{\text{watched}} = i) \rightarrow \text{content}_{w_i}) \\
& \wedge (\overline{\text{watched}} > n) \rightarrow \text{content}_\Box \\
& \wedge \bigwedge_{a \in \Sigma_\Box} \bigwedge_{a' \in \Sigma_\Box \setminus \{a\}} \neg(\text{content}_a \wedge \text{content}_{a'})
\end{aligned}$$

Note: watched tape cell unspecified





# Reduction: Operators

## Operators

One operator for each transition rule $\delta(q, a) = (q', a', \Delta)$:

- precondition:
  $\text{state}_q$
  $\wedge ((\overline{\text{head}} = \overline{\text{watched}}) \rightarrow \text{content}_a)$
  If $\Delta = -1$, conjoin with $\overline{\text{head}} > 0$.
  If $\Delta = +1$, conjoin with $\overline{\text{head}} < 2^{p(n)} - 1$.
- effect:
  $\neg\text{state}_q$
  $\wedge \text{state}_{q'}$
  $\wedge (\overline{\text{head}} := \overline{\text{head}} + \Delta)$
  $\wedge ((\overline{\text{head}} = \overline{\text{watched}}) \rhd (\neg\text{content}_a \wedge \text{content}_{a'}))$

## Reduction: Goal

Goal
$G = \bigvee_{q \in Q_Y} \text{state}_q$





## There exists a plan iff there exists an accepting computation

Proof.
Assume that there exists an accepting computation and consider the corresponding conformant plan. The belief state contains one world state for each (watched) tape cell. Consequently, each operator is applicable and changes the appropriate tape contents in the watched tape cell in the corresponding world state. The TM state and head position is changed in all world states. Hence, the last operator switches to an accepting TM state and the plan reaches the goal.

Conversely, . . . □





## There exists a plan iff there exists an accepting computation [continued]

Continued.
Conversely, assume there exists a plan that reaches the goal and that this plan does not correspond to an accepting computation. Consider the first deviating operator. If the TM state is wrong, then the operator is not applicable. Similarly, if the symbol is wrong, then there is one world state in the belief state where the watched tape cell is the cell under the head. So the operator is not applicable. Hence it cannot be a successful plan.

So, there exists a plan iff there exists an accepting computation

□





## Summary

- Conformant planning is EXPSPACE-hard, i.e. harder than nondeterministic planning under full observability
- Proof is done using the "watched tape cell" trick
- The TM tape is simulated using the different world states in a belief state
- Reduction can be extended to cover the simpler case, where the initial state is described by a CNF formula and all conditions (including the goal) are conjunctions of positive atoms (Conformant-FF).