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Reminder: Restrictions on observability

Let \( \langle A, I, O, G, V \rangle \) be a problem instance in nondeterministic planning.

1. If \( A = V \), the problem instance is full observable.
2. If \( V = \emptyset \), the problem instance is unobservable.
3. If there are no restrictions on \( V \) then the problem instance is partially observable.

Planning without observability: conformant planning

- Here we consider the second special case of planning with partial observability: planning without observability.
- Plans are sequences of actions because observations are not possible, actions cannot depend on the nondeterministic events or uncertain initial state, and hence the same actions have to be taken no matter what happens.
- Techniques needed for planning without observability can often be generalized to the general partially observable case.
Why acting without observation?

- Conformant planning is like planning to act in an environment while you are blind and deaf.
- Observations could be expensive or it is preferable to have a simple plan.
- Example: Finding synchronization sequences in hardware circuits
- Example: Initializing a system consisting of many components that are in unknown states.
- Internal motivation: try to understand the unobservable case so that one can better deal with the more complicated partially observable case.

Belief states and the belief space

- The current state is not in general known during plan execution. Instead, a set of possible current states is known.
- The set of possible current states forms the belief state.
- The set of all belief states is the belief space.
- If there are \( n \) states and none of them can be observationally distinguished from another, then there are \( 2^n - 1 \) belief states.

The belief space

1. Let \( B \) be a belief state (a set of states).
2. Operator \( o \) is executable in \( B \) if it is executable in every \( s \in B \).
3. When \( o \) is executed, possible next states are \( T = \text{img}_o(B) \).
4. Belief states can be succinctly represented using Boolean formulae or BDDs.
The belief space

Example

- A robot without any sensors, anywhere in a room of size 7 × 8.
- Actions: go North, South, East, West; if no way, just stay where you are
- Plan for getting out: 6 × West, 7 × North, 1 × East, 1 × North
- On the next slides we depict one possible location of the robot (●) and the change in the belief state at every execution step by gray fields.
The belief space

Sorting networks

Sorting networks consist of comparator-swapper elements that compare the values of two inputs and output them sorted: if first input is bigger than the second, then they are swapped, otherwise the outputs are the inputs. A sorting network for $n$ inputs should sort any input sequence.

Theorem

If a sorting network correctly sorts any sequence of binary digits 0 and 1, then it correctly sorts any input sequence.

3-input sorting networks can be formalized as a succinct transition system $\langle A, I, O, G, V \rangle$ where:

$$
A = \{a_0, a_1, a_2\}
$$

$$
I = \top
$$

$$
O = \{o_{01}, o_{02}, o_{12}\}
$$

$$
G = (a_0 \rightarrow a_1) \land (a_1 \rightarrow a_2)
$$

$$
o_{01} = \langle T, (a_0 \land \neg a_1) \triangleright (\neg a_0 \land a_1) \rangle
$$

$$
o_{02} = \langle T, (a_0 \land \neg a_2) \triangleright (\neg a_0 \land a_2) \rangle
$$

$$
o_{12} = \langle T, (a_1 \land \neg a_2) \triangleright (\neg a_1 \land a_2) \rangle
$$
The belief space

Sorting networks

A plan for the 3-input sorting network is $o_{12}$, $o_{02}$, $o_{01}$.
The initial states are 000, 001, 010, 011, 100, 101, 110, 111.
The goal states are 000, 001, 011, 111.
The belief state evolves as follows.

000, 001, 010, 011, 100, 101, 110, 111 initially
000, 001, 010, 011, 100, 101, 110, 111 after $o_{12}$
000, 001, 010, 011, 100, 101, 110, 111 after $o_{02}$
000, 001, 010, 011, 100, 101, 110, 111 after $o_{01}$

Algorithms for unobservable problems

1. Find an operator sequence $o_1, \ldots, o_n$ that reaches a state satisfying $G$ starting from any state satisfying $I$.
2. $o_1$ must be applicable in all states $B_0 = \{s \in \mathcal{S} | s \models I\}$ satisfying $I$.
   $o_2$ must be applicable in all states in $B_1 = \text{img}_{o_1}(B_0)$.
   $o_i$ must be applicable in all states in $B_i = \text{img}_{o_i}(B_{i-1})$ for all $i \in \{1, \ldots, n\}$.
Terminal states must be goal states: $B_n \subseteq \{s \in \mathcal{S} | s \models G\}$.

Heuristic search in belief space

progression/regression + heuristic search ($A^*$, $IDA^*$, simulated annealing, ...)

Heuristics:
- heuristic 1: backward distances (for forward search)
- heuristic 2: cardinality of belief state (for both forward and backward search)
Distance heuristics

Use backward distances of states as a heuristic:

\[
D_0 = G \\
D_{i+1} = D_i \cup \bigcup_{o \in O} \text{spreimg}_o(D_i) \text{ for all } i \geq 1
\]

A lower bound on plan length for belief state \( B \) is \( j \) if \( B \subseteq D_j \) and \( B \not\subseteq D_{j-1} \) for \( j \geq 1 \).

This is an admissible heuristic (does not overestimate the distance).

Cardinality heuristics

- Backward search: Prefer operators that increase the size of the belief state, i.e. find a plan suffix that reaches a goal state from more starting states.
- Forward search: Prefer operators that decrease the size of the belief state, i.e. reduce the uncertainty about the current state and make reaching goals easier.

For the room navigation example this heuristic works very well until the size of the belief state is 1.

- This heuristic is not admissible.
- Computing the cardinality of a belief state from its BDD representation takes linear time. (Propositional logic in general: problem is NP-hard.)
- Backward search with the cardinality heuristic seems to work particularly well on the examples from the literature.

Lazy representations

- Conformant-FF: Lazy representation of belief states
  - Instead of computing the belief states (explicitly or symbolically), one could just store the representation of the initial state and the plan (as propositional formula) so far – a lazy representation
  - Works particularly well when all conditions in STRIPS-form, i.e., conjunctions of atoms and deterministic operators (can be extended)
  - Necessarily true atoms at each point in the plan can be computed using one UNSAT-call
  - The FF heuristic \( h_{\text{FF}} \) can be extended to deal with belief state planning by using an unsound approximation of the propositional formula.

Representing the plan as a CNF formula

- Let \( \pi \) be the plan \( \langle o_1, \ldots, o_n \rangle \).
- The atoms \( o_t \) are indexed by the time point \( t \), i.e., \( o_t(t) \).
- The initial belief state (at time point 0) is presented as a CNF formula over literals indexed with 0.
- Given we have a representation for the plan up time point \( t \), we extend the formula as follows:
  1. Effect axioms: For every effect \( e \) (in normal form) of \( o_{t+1} \) with \( e = ((a_1 \land \ldots \land a_m) \Rightarrow l) \), we insert the formula
     \[
     \neg a_1(t) \lor \ldots \lor \neg a_m(t) \lor l(t + 1)
     \]
  2. Frame axioms: for every atom \( a \), let \( e_1, \ldots, e_l \) be the effects that contain \( a \) as a negative atomic effect; for every tuple \( a_1, \ldots, a_n \) such that \( a_i \) is a part of \( e_j \)'s effect condition, we insert
     \[
     \neg a(t) \lor a_1(t) \lor \ldots \lor a_n(t) \lor a(t + 1).
     \]

    Similarly for positive atomic effects!
Example

- **Operator** \( o = \langle T, (a \land c) \triangleright \neg a \rangle \)
- **Initial belief state** = \( a \lor b, \neg a \lor \neg b, c \)
- **Plan** \( \pi = \langle o \rangle \)
- **Construction:**
  1. **Initial state:** \( a(0) \lor b(0), \neg a(0) \lor \neg b(0), c(0) \)
  2. **Effect axiom:** \( \neg a(0) \lor \neg c(0) \lor \neg a(1) \)
  3. **Simple positive frame axioms:** \( \neg b(0) \lor b(1), \neg c(0) \lor c(1) \)
  4. **Complex positive frame axioms:** \( \neg a(0) \lor a(0) \lor a(1), \neg a(0) \lor c(0) \lor a(1) \)
  5. **Simple negative frame axioms:** \( a(0) \lor \neg a(1), b(0) \lor \neg b(1), c(0) \lor \neg c(1) \)

Computing the necessary true and false atoms

- In order to check whether an operator is **applicable** or the goal has been reached, one need to know, whether a set of atoms is **necessarily true**.
- Simply add \( \neg a_i(t) \) and check for satisfiability. If it is unsatisfiable, \( a_i \) is necessarily true at time point \( t \)
- Necessarily true and false atoms can be cached to speed up reasoning.
- **Problem:** Designing a search heuristic in belief space!

Extending the \( h_{\text{FF}} \) heuristics

- **Reminder:** FF computes the heuristic estimate by ignoring negative effects and trying to generate near-optimal plan for this relaxation.
- We do the same here and additionally . . .
- We over-approximate the clause set by reducing all clauses to two-literal clauses -- randomly
- This theory is stronger, i.e. it is complete and most probably unsound
- Satisfiability can be solved in linear time

Summary

- **Conformant planning** is planning in a non-deterministic context without observation
- The search space is the belief space, the space of all belief sets.
- Techniques from classical planning can be lifted to belief space search
- BDDs are one possibility to implement this kind of search and model counting appears to be a reasonable heuristics
- Another possibility is lazy representation of plans as in Conformant-FF