Principles of AI Planning

Complexity of nondeterministic planning with full observability

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Overview

- Similar to the earlier analysis of deterministic planning, we will now study the computational complexity of nondeterministic planning with full observability.
- We consider the case of strong planning.
- The results for strong cyclic planning are identical.

As usual, the main motivation for such a study is to determine the limit of what is possible algorithmically: Should we try to develop a polynomial algorithm?

Comparison to deterministic planning

- The basic proof idea is very similar to the PSPACE-completeness proof for deterministic planning.
- The main difference is that we consider alternating Turing Machines (ATMs) instead of deterministic Turing Machines (DTMs) in the reduction.
- Due to the similarity to the earlier proof, we first review some of the concepts introduced in the earlier lecture.

Alternating Turing Machines

Definition: Alternating Turing Machine

Alternating Turing Machine (ATM) $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$:

1. input alphabet Σ and blank symbol $\Box \notin \Sigma$
   - alphabets always non-empty and finite
   - tape alphabet $\Sigma_\Box = \Sigma \cup \{\Box\}$
2. finite set Q of internal states with initial state $q_0 \in Q$
3. state labeling $l : Q \rightarrow \{\mathsf{Y}, \mathsf{N}, \exists, \forall\}$
   - accepting, rejecting, existential, universal states Q_Y, Q_N, $Q_\exists$, $Q_\forall$
   - terminal states $Q_\star = Q_\mathsf{Y} \cup Q_\mathsf{N}$
   - nonterminal states $Q' = Q_\exists \cup Q_\forall$
4. transition relation $\delta \subseteq (Q' \times \Sigma_\Box) \times (Q \times \Sigma_\Box \times \{-1, +1\})$

Turing Machine configurations

Let $M = \langle \Sigma, \square, Q, q_0, l, \delta \rangle$ be an ATM.

Definition: Configuration

A configuration of M is a triple $(w, q, x) \in \Sigma_\square^* \times Q \times \Sigma_\square^+$.

- w: tape contents before tape head
- q: current state
- x: tape contents after and including tape head

Turing Machine transitions

Let $M = \langle \Sigma, \square, Q, q_0, l, \delta \rangle$ be an ATM.

Definition: Yields relation

A configuration c of M yields a configuration c' of M, in symbols $c \vdash c'$, as defined by the following rules, where $a, a', b \in \Sigma_\square$, $w, x \in \Sigma_\square^*$, $q, q' \in Q$ and $((q, a), (q', a', \Delta)) \in \delta$:

$$
\begin{aligned}
(w, q, ax) &\vdash (wa', q', x) && \text{if } \Delta = +1, |x| \geq 1 \\
(w, q, a) &\vdash (wa', q', \square) && \text{if } \Delta = +1 \\
(wb, q, ax) &\vdash (w, q', ba'x) && \text{if } \Delta = -1 \\
(\epsilon, q, ax) &\vdash (\epsilon, q', \square a'x) && \text{if } \Delta = -1
\end{aligned}
$$

Acceptance (space)

Let $M = \langle \Sigma, \square, Q, q_0, l, \delta \rangle$ be an ATM.

Definition: Acceptance (space)

Let $c = (w, q, x)$ be a configuration of M.

- M accepts $c = (w, q, x)$ with $q \in Q_{\mathsf{Y}}$ in space n iff $|w| + |x| \leq n$.
- M accepts $c = (w, q, x)$ with $q \in Q_{\exists}$ in space n iff M accepts some c' with $c \vdash c'$ in space n.
- M accepts $c = (w, q, x)$ with $q \in Q_{\forall}$ in space n iff M accepts all c' with $c \vdash c'$ in space n.

Accepting words and languages

Let $M = \langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ be an ATM.

Definition: Accepting words

M accepts the word $w \in \Sigma^*$ in space $n \in \mathbb{N}_0$
iff M accepts (ϵ, q_0, w) in space n.

- Special case: M accepts ϵ in time (space) $n \in \mathbb{N}_0$ iff M accepts $(\epsilon, q_0, \Box)$ in time (space) n.

Definition: Accepting languages

Let $f : \mathbb{N}_0 \to \mathbb{N}_0$.
M accepts the language $L \subseteq \Sigma^*$ in space f
iff M accepts each word $w \in L$ in space $f(|w|)$,
and M does not accept any word $w \notin L$.

Alternating space complexity

Definition: ASPACE, APSPACE

Let $f : \mathbb{N}_0 \to \mathbb{N}_0$.

Complexity class ASPACE(f) contains all languages accepted in space f by some ATM.

Let $\mathcal{P}$ be the set of polynomials $p : \mathbb{N}_0 \to \mathbb{N}_0$.

$$\text{APSPACE} := \bigcup_{p \in \mathcal{P}} \text{ASPACE}(p)$$

Standard complexity classes relationships

Theorem

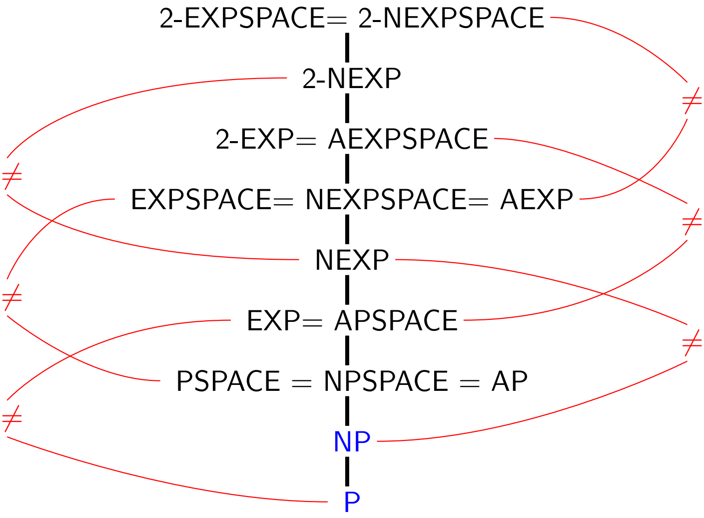
$$\begin{array}{rcccl}
\mathrm{P} & \subseteq & \mathrm{NP} & \subseteq & \mathrm{AP} \\
\mathrm{PSPACE} & \subseteq & \mathrm{NPSPACE} & \subseteq & \mathrm{APSPACE} \\
\mathrm{EXP} & \subseteq & \mathrm{NEXP} & \subseteq & \mathrm{AEXP} \\
\mathrm{EXPSPACE} & \subseteq & \mathrm{NEXPSPACE} & \subseteq & \mathrm{AEXPSPACE} \\
\mathrm{2\text{-}EXP} & \subseteq & \dots & &
\end{array}$$

The power of alternation

Theorem (Chandra et al. 1981)

$$
\begin{aligned}
\mathrm{AP} &= \mathrm{PSPACE} \\
\mathrm{APSPACE} &= \mathrm{EXP} \\
\mathrm{AEXP} &= \mathrm{EXPSPACE} \\
\mathrm{AEXPSPACE} &= \text{2-EXP}
\end{aligned}
$$

The hierarchy of complexity classes



The strong planning problem

StrongPlanEx (strong plan existence)

Given: nondeterministic planning task $\langle A, I, O, G, V \rangle$ with full observability ($A = V$)

Question: Is there a strong plan for the task?

- We do not consider a nondeterministic analog of the bounded plan existence problem (PlanLen).

Proof idea

- We will prove that StrongPlanEx is EXP-complete.
- We already know that the problem belongs to EXP, because we have presented a dynamic programming algorithm that generates strong plans in exponential time.
- We prove hardness for EXP by providing a generic reduction for alternating Turing Machines with polynomial space and use Chandra et al.'s theorem showing APSPACE $=$ EXP.

Reduction
Overview

- For a fixed polynomial p, given ATM M and input w, generate planning task which is solvable by a strong plan iff M accepts w in space $p(|w|)$.
- For simplicity, restrict to ATMs which never move to the left of the initial head position (no loss of generality).
- Existential states of the ATM are modeled by states of the planning task where there are several applicable operators to choose from.
- Universal states of the ATM are modeled by states of the planning task where there is a single applicable operator with a nondeterministic effect.

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Reduction: state variables

Let p be the space-bound polynomial.

Given ATM $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

State variables

- state_q for all $q \in Q$
- head_i for all $i \in X \cup \{0, p(n) + 1\}$
- $\mathsf{content}_{i,a}$ for all $i \in X$, $a \in \Sigma_\Box$

Reduction: initial state

Let p be the space bound polynomial.

Given ATM $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Initial state formula

Specify a unique initial state.

Initially true:

- state_{q_0}
- head_1
- $\text{content}_{i,w_i}$ for all $i \in \{1, \dots, n\}$
- $\text{content}_{i,\Box}$ for all $i \in X \setminus \{1, \dots, n\}$

Initially false:

- all others

Reduction: goal

Let p be the space bound polynomial.

Given ATM $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Goal

$\bigvee_{q \in Q_{\mathsf{Y}}} \mathsf{state}_q$

- Without loss of generality, we can assume that Q_{Y} is a singleton set so that we do not need a disjunctive goal.
- This way, the hardness result also holds for a restricted class of planning tasks ("nondeterministic STRIPS").

Reduction: operators

Let p be the space bound polynomial.

Given ATM $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Operators

For $q, q' \in Q$, $a, a' \in \Sigma_\Box$, $\Delta \in \{-1, +1\}$, $i \in X$, define

- $\mathsf{pre}_{q,a,i} = \mathsf{state}_q \wedge \mathsf{head}_i \wedge \mathsf{content}_{i,a}$
- $\mathsf{eff}_{q,a,q',a',\Delta,i} = \neg\mathsf{state}_q \wedge \neg\mathsf{head}_i \wedge \neg\mathsf{content}_{i,a} \wedge \mathsf{state}_{q'} \wedge \mathsf{head}_{i+\Delta} \wedge \mathsf{content}_{i,a'}$
  - If $q = q'$, omit the effects $\neg\mathsf{state}_q$ and $\mathsf{state}_{q'}$.
  - If $a = a'$, omit the effects $\neg\mathsf{content}_{i,a}$ and $\mathsf{content}_{i,a'}$.

Reduction: operators (continued)

Let p be the space bound polynomial.

Given ATM $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Operators (ctd.)

For existential states $q \in Q_\exists$, $a \in \Sigma_\Box$, $i \in X$:
Let $(q'_j, a'_j, \Delta_j)_{j \in \{1,\dots,k\}}$ be those triples with $((q, a), (q'_j, a'_j, \Delta_j)) \in \delta$.

For each $j \in \{1, \dots, k\}$, introduce one operator:

- precondition: $\mathrm{pre}_{q,a,i}$
- effect: $\mathrm{eff}_{q,a,q'_j,a'_j,\Delta_j,i}$

Reduction: operators (continued)

Let p be the space bound polynomial.

Given ATM $\langle \Sigma, \Box, Q, q_0, l, \delta \rangle$ and input $w_1 \dots w_n$, define relevant tape positions $X = \{1, \dots, p(n)\}$.

Operators (ctd.)

For universal states $q \in Q_\forall$, $a \in \Sigma_\Box$, $i \in X$:
Let $(q'_j, a'_j, \Delta_j)_{j \in \{1,\dots,k\}}$ be those triples with $((q, a), (q'_j, a'_j, \Delta_j)) \in \delta$.

Introduce only one operator:

- precondition: $\mathsf{pre}_{q,a,i}$
- effect: $\mathsf{eff}_{q,a,q'_1,a'_1,\Delta_1,i} | \dots | \mathsf{eff}_{q,a,q'_k,a'_k,\Delta_k,i}$

EXP-completeness of strong planning with full observability

Theorem (Rintanen)

STRONGPLANEX *is EXP-complete.*

This is true even if we only allow operators in unary nondeterminism normal form where all deterministic sub-effects and the goal satisfy the STRIPS restriction and if we require a deterministic initial state.

Proof.

Membership in EXP has been shown by providing exponential-time algorithms that generate strong plans (and decide if one exists as a side effect).

Hardness follows from the previous generic reduction for ATMs with polynomial space bound and Chandra et al.'s theorem. □

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Summary

- Nondeterministic planning is harder than deterministic planning.
- In particular, it is EXP-complete in the fully observable case, compared to the PSPACE-completeness of deterministic planning.
- The hardness result already holds if the operators and goals satisfy some fairly strong syntactic restrictions and there is a unique initial state.
- The introduction of nondeterministic effects corresponds to the introduction of alternation in Turing Machines.
- Later, we will see that restricted observability has an even more dramatic effect on the complexity of the planning problem.