# Principles of AI Planning

January 17th, 2007 — Strong nondeterministic planning with full observability

# Principles of AI Planning

## Strong nondeterministic planning with full observability

Malte Helmert    Bernhard Nebel

Albert-Ludwigs-Universität Freiburg

January 17th, 2007

# Strong planning with full observability

We will first consider one of the simplest cases of nondeterministic planning by restricting attention to:

- fully observable planning tasks and
- strong plans.

In this lesson, planning task always means fully observable nondeterministic planning task.

# Memoryless strategies

## Definition

As noted previously, in the fully observable case, we can use simpler notions of strategies and plans.

### Definition

Let $S$ be the set of states of a planning task $\mathcal{T}$.
A memoryless strategy for $\mathcal{T}$ is a partial function $\pi : S \to O$ such that $\pi(s)$ is applicable wherever $\pi(s)$ is defined.

### Execution of a memoryless strategy

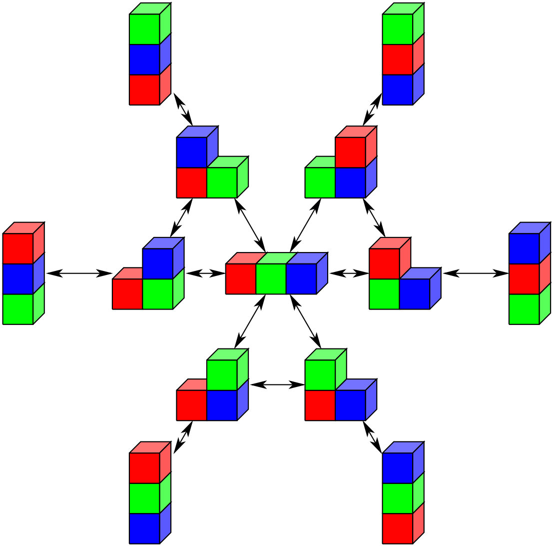
1. Determine the current state $s$ (full observability!).
2. If $\pi(s)$ is not defined then terminate execution.
   (If $s$ is a goal state, then $\pi(s)$ should not be defined so that the execution terminates.)
3. Execute action $\pi(s)$.
4. Repeat from first step.

# Memoryless plans

- Memoryless strategies can be straightforwardly translated to strategies as introduced in the previous lesson.
- We do not discuss this.
- Following the definitions from the previous lesson, we can introduce concepts such as weak memoryless plans, strong memoryless plans etc.

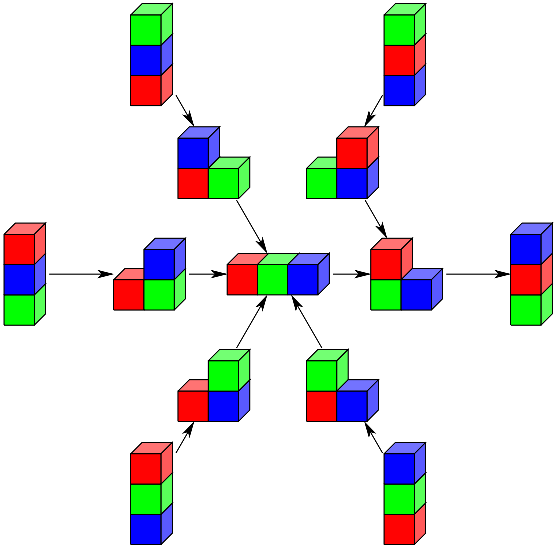
# Memoryless plans

## Memoryless plan (deterministic operators, uncertain initial state)



# Memoryless plans

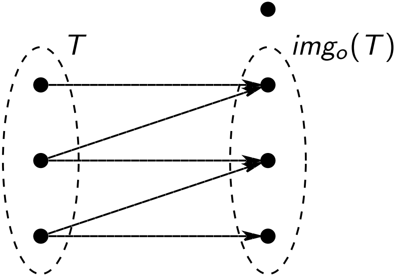
## Memoryless plan (deterministic operators, uncertain initial state)



# Images

## Image

The image of a set $T$ of states with respect to an operator $o$ is the set of those states that can be reached by executing $o$ in a state in $T$.



# Images

## Formal definition

### Definition (Image of a state)

$img_o(s) = \{s' \in S | sos'\}$

### Definition (Image of a set of states)

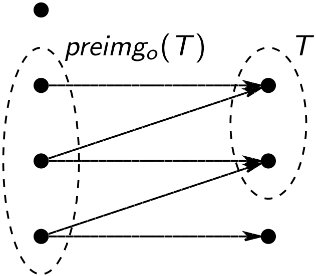
$img_o(T) = \bigcup_{s \in T} img_o(s)$

- Observe that $img_o(T) = app_o(T)$, where $T$ is a belief state. We avoid the term "belief state" in this lesson because the intuition behind this term is wrong for fully observable planning – here, we consider sets of states together for algorithmic or efficiency reasons, not because they cannot be distinguished.

# Weak preimages

## Weak preimage

The weak preimage of a set $T$ of states with respect to an operator $o$ is the set of those states from which a state in $T$ can be reached by executing $o$.



# Weak preimages

## Formal definition

**Definition (Weak preimage of a state)**

$preimg_o(s') = \{s \in S | sos'\}$

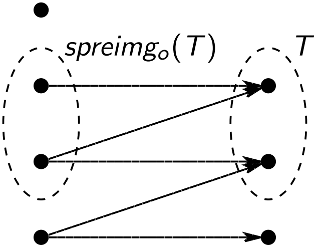
**Definition (Weak preimage of a set of states)**

$preimg_o(T) = \bigcup_{s \in T} preimg_o(s)$.

# Strong preimages

## Strong preimage

The strong preimage of a set $T$ of states with respect to an operator $o$ is the set of those states from which a state in $T$ is always reached when executing $o$.



# Strong preimages

## Formal definition

**Definition (Strong preimage of a set of states)**

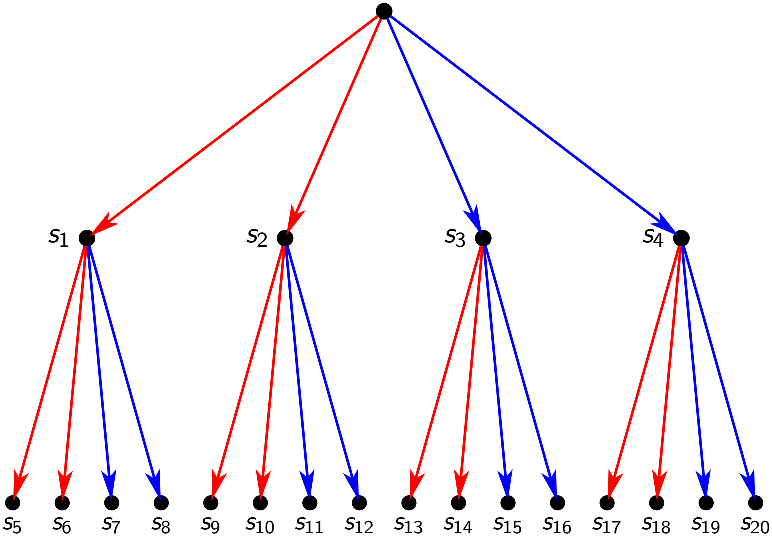
$spreimg_o(T) = \{s \in S \mid \exists s' \in T : sos', img_o(s) \subseteq T\}$

# Algorithms for fully observable problems

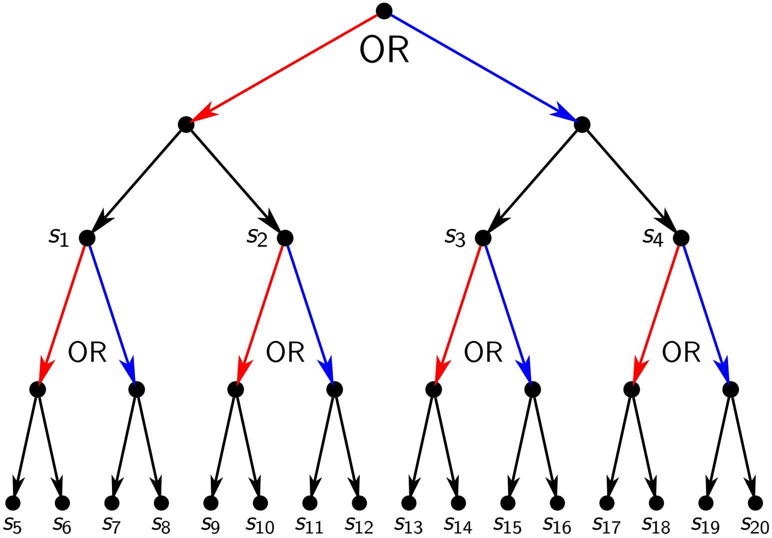
1. Heuristic search (forward)
   Strong planning can be viewed as AND-OR search.
   OR nodes: Choice between operators
   AND nodes: Nondeterministically reached state
   Heuristic AND-OR search algorithms:
   AO*, B*, Proof Number Search, ...
2. Dynamic programming (backward)
   Compute operator/distance/value for a state based on the operators/distances/values of its all successor states.
   2.1 0 actions needed for goal states.
   2.2 If states with $i$ actions to goals are known, states with $\leq i+1$ actions to goals can be easily identified.

   Automatic reuse of already found plan suffixes.

# AND-OR search



# AND-OR search



# Dynamic programming

## Planning by dynamic programming

If for all successors of state $s$ with respect to operator $o$ a plan exists, assign operator $o$ to $s$.

Base case $i = 0$: In goal states there is nothing to do.

Inductive case $i \geq 1$: If there is $o \in O$ such that for all $s' \in img_o(s)$, the state $s'$ is a goal state or $\pi(s')$ was assigned in an earlier iteration, then assign $\pi(s) = o$.

## Backward distances

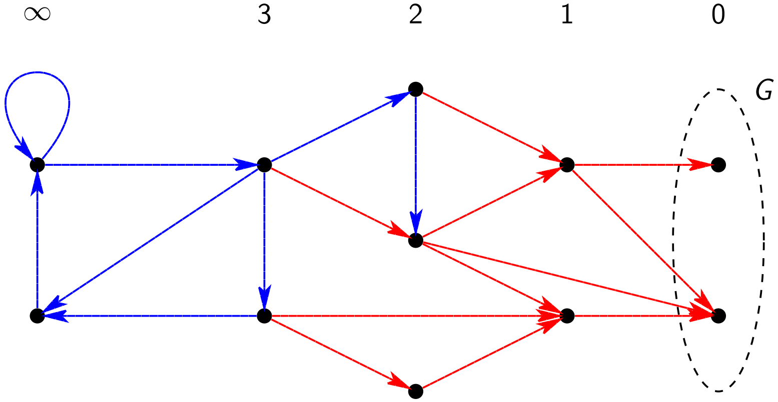
If $s$ is assigned a value on iteration $i \geq 1$, then the backward distance of $s$ is $i$.

The dynamic programming algorithm essentially computes the backward distances of states.

# Backward distances

## Example

distance to $G$



# Backward distances

## Definition of distance sets

### Definition

Let $G$ be a set of states and $O$ a set of operators.
The backward distance sets $D_i^{bwd}$ for $G$ and $O$ consist of those states for which there is a guarantee of reaching a state in $G$ with at most $i$ operator applications using operators in $O$:

$$D_0^{bwd} := G$$
$$D_i^{bwd} := D_{i-1}^{bwd} \cup \bigcup_{o \in O} spreimg_o(D_{i-1}^{bwd}) \text{ for all } i \geq 1$$

# Backward distances

## Definition

### Definition

Let $G$ be a set of states and $O$ a set of operators, and let $D_0^{bwd}, D_1^{bwd}, \dots$ be the backward distance sets for $G$ and $O$. Then the backward distance of a state $s$ for $G$ and $O$ is

$$\delta_G^{bwd}(s) = \begin{cases} 0 & \text{if } s \in G \\ i & \text{if } s \in D_i^{bwd} \setminus D_{i-1}^{bwd} \text{ for any } i \in \mathbb{N}_1 \\ \infty & \text{otherwise} \end{cases}$$

# Strong memoryless plans based on distances

Let $\mathcal{T} = \langle A, I, O, G, V \rangle$ be a planning task with state set $S$.

## Extraction of a strong memoryless plan from distance sets

1. Let $S' \subseteq S$ be those states having a finite backward distance for $G$ and $O$.
2. Let $s \in S'$ be a state with distance $i = \delta_G^{bwd}(s) \geq 1$.
3. Assign to $\pi(s)$ any operator $o \in O$ such that $img_o(s) \subseteq D_{i-1}^{bwd}$. Hence $o$ decreases the backward distance by at least one.

Then $\pi$ is a strong plan for $\mathcal{T}$ iff $\{s \in S \mid s \models I\} \subseteq S'$.

Question: What is the worst-case runtime of the algorithm?

Question: What is the best-case runtime of the algorithm if most states have a finite backward distance?

# Making the algorithm a logic-based algorithm

- An algorithm that represents the states explicitly stops being feasible at about $10^8$ or $10^9$ states.
- For planning with bigger transition systems structural properties of the transition system have to be taken advantage of.
- As before, representing state sets as propositional formulae or BDDs often allows taking advantage of the structural properties: a formula or BDD that represents a set of states or a transition relation that has certain regularities may be very small in comparison to the set or relation.
- In the following, we will present a BDD-based algorithm.

# Breadth-first search with progression and state sets

Reminder: Algorithm for the deterministic case

## Progression breadth-first search

**def** bfs-progression($A$, $I$, $O$, $G$):
  $goal := formula\text{-}to\text{-}set(G)$
  $reached := \{I\}$
  **loop**:
    **if** $reached \cap goal \neq \emptyset$:
      **return** solution found
    $new\text{-}reached := reached \cup apply(reached, O)$
    **if** $new\text{-}reached = reached$:
      **return** no solution exists
    $reached := new\text{-}reached$

$\rightsquigarrow$ This can easily be transformed into a regression algorithm.

# Breadth-first search with regression and state sets

Algorithm for the deterministic case

## Regression breadth-first search

**def** bfs-regression($A$, $I$, $O$, $G$):
    $init := I$
    $reached := formula\text{-}to\text{-}set(G)$
    **loop**:
        **if** $init \in reached$:
            **return** solution found
        $new\text{-}reached := reached \cup apply^{-1}(reached, O)$
        **if** $new\text{-}reached = reached$:
            **return** no solution exists
        $reached := new\text{-}reached$

- This algorithm is very similar to the dynamic programming algorithm for the nondeterministic case!

# Breadth-first search with regression and state sets

## Algorithm for the nondeterministic case

### Regression breadth-first search

**def** bfs-regression($A$, $I$, $O$, $G$):
    $\mathit{init} := \mathit{formula\text{-}to\text{-}set}(I)$
    $\mathit{reached} := \mathit{formula\text{-}to\text{-}set}(G)$
    **loop**:
        **if** $\mathit{init} \subseteq \mathit{reached}$:
            **return** solution found
        $\mathit{new\text{-}reached} := \mathit{reached} \cup \bigcup_{o \in O} \mathit{spreimg}_o(\mathit{reached})$
        **if** $\mathit{new\text{-}reached} = \mathit{reached}$:
            **return** no solution exists
        $\mathit{reached} := \mathit{new\text{-}reached}$

- How do we define *spreimg* with set-theoretic (BDD) operations?

# Computing strong preimages

## Strong preimages

$$\begin{aligned}
spreimg_o(T) &= \{s \in S \mid \exists s' \in T : sos', img_o(s) \subseteq T\} \\
&= \{s \in S \mid (\exists s' \in S : s' \in T \wedge sos') \wedge \\
&\qquad \{s' \in S \mid sos'\} \subseteq T\} \\
&= \{s \in S \mid (\exists s' \in S : s' \in T \wedge sos') \wedge \\
&\qquad (\forall s' \in S : sos' \rightarrow (s' \in T))\} \\
&= \{s \in S \mid (\exists s' \in S : s' \in T \wedge sos') \wedge \\
&\qquad (\neg \exists s' \in S : sos' \wedge s' \notin T)\}
\end{aligned}$$

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \land sos') \land (\neg\exists s' \in S : sos' \land s' \notin T)\}$$

## Strong preimages with BDDs

**def** rename-A-to-A'($B$):
    **for each** $a \in A$:
        $B := \mathit{bdd\text{-}rename}(B, a, a')$
    **return** $B$

**def** forget-A'($B$):
    **for each** $a \in A$:
        $B := \mathit{bdd\text{-}forget}(B, a')$
    **return** $B$

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \land sos') \land (\neg\exists s' \in S : sos' \land s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

$s$'-$in$-$T$ := $rename$-$A$-$to$-$A$'($T$)

$s$'-$not$-$in$-$T$ := $bdd$-$complement$($s$'-$in$-$T$)

$B_1$ := $forget$-$A$'($bdd$-$intersection$($s$'-$in$-$T$, $T_A(o)$))

$B_2$ := $forget$-$A$'($bdd$-$intersection$($T_A(o)$, $s$'-$not$-$in$-$T$))

**return** $bdd$-$intersection$($B_1$, $bdd$-$complement$($B_2$))

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \land sos') \land (\neg\exists s' \in S : sos' \land s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

  *s'-in-T* := *rename-A-to-A'*($T$)

  *s'-not-in-T* := *bdd-complement*(*s'-in-T*)

  $B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))

  $B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))

  **return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \wedge sos') \wedge (\neg\exists s' \in S : sos' \wedge s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

*s'-in-T* := *rename-A-to-A'*($T$)
*s'-not-in-T* := *bdd-complement(s'-in-T)*
$B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))
$B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))
**return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \land sos') \land (\neg \exists s' \in S : sos' \land s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

   *s'-in-T* := *rename-A-to-A'*($T$)
   *s'-not-in-T* := *bdd-complement*(*s'-in-T*)
   $B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))
   $B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))
   **return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \wedge sos') \wedge (\neg\exists s' \in S : sos' \wedge s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

*s'-in-T* := *rename-A-to-A'*($T$)

*s'-not-in-T* := *bdd-complement*(*s'-in-T*)

$B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))

$B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))

**return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

# Computing strong preimages with BDDs operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \wedge sos') \wedge (\neg \exists s' \in S : sos' \wedge s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

*s'-in-T* := *rename-A-to-A'*($T$)
*s'-not-in-T* := *bdd-complement*(*s'-in-T*)
$B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))
$B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))
**return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

# Computing strong preimages with BDD operations

$$\mathit{spreimg}_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \land sos') \land (\neg\exists s' \in S : sos' \land s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):
  *s'-in-T* := *rename-A-to-A'*($T$)
  *s'-not-in-T* := *bdd-complement*(*s'-in-T*)
  $B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))
  $B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))
  **return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \wedge sos') \wedge (\neg\exists s' \in S : sos' \wedge s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

  *s'-in-T* := *rename-A-to-A'*($T$)

  *s'-not-in-T* := *bdd-complement*(*s'-in-T*)

  $B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))

  $B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))

  **return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \land sos') \land (\neg\exists s' \in S : sos' \land s' \notin T)\}$$

## Strong preimages with BDDs

```
def strong-preimage(o, T):
    s'-in-T := rename-A-to-A'(T)
    s'-not-in-T := bdd-complement(s'-in-T)
    B1 := forget-A'(bdd-intersection(s'-in-T, T_A(o)))
    B2 := forget-A'(bdd-intersection(T_A(o), s'-not-in-T))
    return bdd-intersection(B1, bdd-complement(B2))
```

# Computing strong preimages with BDD operations

$$spreimg_o(T) = \{s \in S \mid (\exists s' \in S : s' \in T \land sos') \land (\neg\exists s' \in S : sos' \land s' \notin T)\}$$

## Strong preimages with BDDs

**def** strong-preimage($o$, $T$):

  *s'-in-T* := *rename-A-to-A'*($T$)
  *s'-not-in-T* := *bdd-complement*(*s'-in-T*)
  $B_1$ := *forget-A'*(*bdd-intersection*(*s'-in-T*, $T_A(o)$))
  $B_2$ := *forget-A'*(*bdd-intersection*($T_A(o)$, *s'-not-in-T*))
  **return** *bdd-intersection*($B_1$, *bdd-complement*($B_2$))

Are we done?

No, because we have not yet shown how to compute $T_A(o)$ for nondeterministic operators.

# Transition formula for nondeterministic operators

The formula $\tau_A(o)$ (on which the BDD/relation $T_A(o)$ is based) must express

- the conditions for applicability of $o$,
- how $o$ changes state variables, and
- which state variables $o$ does not change.

A significant difficulty lies in the third requirement because different variables may be affected depending on nondeterministic choices.

# Normal forms for nondeterministic operators

- In deterministic planning, we translated effects to normal form to express them in propositional logic.
- For nondeterministic effects, there is no (simple) normal form with all the nice properties of deterministic operator normal form:
  - expressiveness (all effects are convertible to normal form)
  - efficient computability
  - simple representation in propositional logic
- We will thus introduce different normal forms which have a subset of these properties.

# Unary nondeterminism normal form

## Definition

### Definition

An effect $e$ is in unary nondeterminism normal form iff

- $e$ is deterministic and in normal form, or
- $e = e_1 \,|\, \ldots \,|\, e_n$ where each $e_i$ is deterministic and in normal form.

- What about simple representation, expressiveness and efficient computability?

# Unary nondeterminism normal form

## Simple representation

### Recall: $\tau_A(o)$ for deterministic operators $o = \langle c, e \rangle$

$$\tau_A(o) = c \wedge \bigwedge_{a \in A} ((EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))) \leftrightarrow a') \\ \wedge \bigwedge_{a \in A} \neg (EPC_a(e) \wedge EPC_{\neg a}(e))$$

For $o = \langle c, e_1 \mid \dots \mid e_n \rangle$ where each $e_i$ is deterministic:

$$\tau_A(o) = c \wedge \bigvee_{i=1}^{n} \bigwedge_{a \in A} ((EPC_a(e_i) \vee (a \wedge \neg EPC_{\neg a}(e_i))) \leftrightarrow a') \\ \wedge \bigwedge_{i=1}^{n} \bigwedge_{a \in A} \neg (EPC_a(e_i) \wedge EPC_{\neg a}(e_i))$$

# Unary nondeterminism normal form

## Expressiveness and efficient computability

Unary nondeterminism normal form is expressive.
Every nondeterministic effect can be converted by using the following equivalences to raise nondeterminism to the root of the effect:

$$\begin{aligned} c \rhd (e_1 \,|\, \dots \,|\, e_n) &\equiv (c \rhd e_1) \,|\, \dots \,|(c \rhd e_n) \\ (e_1 \,|\, \dots \,|\, e_n) \wedge e' &\equiv (e_1 \wedge e') \,|\, \dots \,|(e_n \wedge e') \\ (e_1 \,|\, \dots \,|\, e_n) \,|\, e'_1 \dots \,|\, e'_m &\equiv e_1 \,|\, \dots \,|\, e_n \,|\, e'_1 \,|\, \dots \,|\, e'_m \end{aligned}$$

and then converting the deterministic subeffects using the standard algorithm.

However, this is not efficiently computable because there are operators for which an exponential growth of operator size is unavoidable ($\rightsquigarrow$ exercises).

# Unary nondeterminism normal form

## Discussion

- Unary nondeterminism normal form is among the simplest possible normal forms. There is only one possible nesting of effect types:
  - atomic effects
  - within conditional effects
  - within conjunctive effects
  - within choice effects
- The price for this simplicity is an exponential blow-up in many cases.
- To avoid this blowup, we will now relax the nesting options somewhat.

# Unary conditionality normal form

## Definition

### Definition

An effect $e$ is in unary conditionality normal form iff for all conditional effects $(c \triangleright e')$ occurring within $e$, the effect $e'$ is atomic.

- Note that conjunctive effects and choice effects may be nested arbitrarily.

# Unary conditionality normal form

## Properties

Unary conditionality normal form is expressive.
Every nondeterministic effect can be converted by using the following equivalences to push conditional effects towards the leaves of the effect:

$$
\begin{aligned}
c \rhd (e_1 \mid \dots \mid e_n) &\equiv (c \rhd e_1) \mid \dots \mid (c \rhd e_n) \\
c \rhd (e_1 \wedge \dots \wedge e_n) &\equiv (c \rhd e_1) \wedge \dots \wedge (c \rhd e_n) \\
c \rhd (c' \rhd e) &\equiv (c \wedge c') \rhd e
\end{aligned}
$$

This is also efficiently computable.

However, for this normal form, there does not appear to be a simple representation in propositional logic.

# Unary conditionality normal form

## Discussion

- Unary conditionality normal form allows too complicated nestings of conjunctive and choice effects.
- This makes it difficult to test, for example, whether there are possible choices that will lead to inconsistent effects.
- For this reason, we will now look into a slightly stricter normal form which is a good compromise between our desiderata.

# Decomposablue unary conditionality normal form

## Scope

### Definition

Define the scope of an effect $e$ as

$$\begin{aligned} scope(a) &= \{a\} \\ scope(\neg a) &= \{a\} \\ scope(c \rhd e) &= scope(e) \\ scope(e_1 \wedge \cdots \wedge e_n) &= scope(e_1) \cup \cdots \cup scope(e_n) \\ scope(e_1 \mid \ldots \mid e_n) &= scope(e_1) \cup \cdots \cup scope(e_n) \end{aligned}$$

# Decomposable unary conditionality normal form

## Definition

### Definition

An effect $e$ is in decomposable unary conditionality (DUC) normal form iff it is in unary conditionality normal form and for all conjunctive effects $(e_1 \wedge \cdots \wedge e_n)$ occurring within $e$, either

- all $e_i$ are deterministic, or
- for all $i \neq j$, $scope(e_i)$ and $scope(e_j)$ are disjoint.

Example: $(a \,|\, b) \wedge (\neg b \,|\, d)$ is not in DUC normal form because variable $b$ occurs in $(a \,|\, b)$ and $(\neg b \,|\, d)$.

- Consistency of effect application can be tested easily:
  The effect is guaranteed to be consistent in state $s$ iff this is the case for each deterministic sub-effect.

# Decomposable unary conditionality normal form

## Properties

- DUC normal form is a special case of unary conditionality normal form and a generalization of unary nondeterminism normal form.
- Because it generalizes unary nondeterminism normal form, it is expressive.
- We do not discuss efficient computability in detail, but only note that in practice, nondeterministic operators can usually be compactly represented in DUC normal form.
- We will now consider the property of simple representation.

# Decomposable unary conditionality normal form

## Representation in propositional logic

### Recall: $\tau_A(o)$ for deterministic operators $o = \langle c, e \rangle$

$$\tau_A(o) = c \land \bigwedge_{a \in A} ((EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))) \leftrightarrow a') \land \bigwedge_{a \in A} \neg(EPC_a(e) \land EPC_{\neg a}(e))$$

For nondeterministic $o = \langle c, e \rangle$ where $e$ is in DUC normal form, this generalizes to:

$$\tau_A(o) = c \land \tau_A^{nd}(e) \land \bigwedge_{e' \in E^{det}} \bigwedge_{a \in A} \neg(EPC_a(e') \land EPC_{\neg a}(e'))$$

where $E^{det}$ is the set of deterministic sub-effects of $e$ and $\tau_A^{nd}(e)$ is defined on the following slide.

# Decomposable unary conditionality normal form

## Representation in propositional logic

We make sure that $\tau_A^{nd}(e)$ describes changed and unchanged variables consistently by expressing changes

- for exactly the same variables $B$ within choice effects and
- for disjoint variables $B$ for (nondeterministic) conjunctive effects.

This gives rise to the following recursive definition:

### Definition

$$
\begin{aligned}
\tau_B^{nd}(e) &= \tau_B(e) \text{ for deterministic effects } e \\
\tau_B^{nd}(e_1 \,|\, \dots \,|\, e_n) &= \tau_B^{nd}(e_1) \vee \dots \vee \tau_B^{nd}(e_n) \\
\tau_B^{nd}(e_1 \wedge \dots \wedge e_n) &= \tau_{scope(e_1)}^{nd}(e_1) \wedge \dots \wedge \tau_{scope(e_n)}^{nd}(e_n) \\
&\quad \wedge \bigwedge_{a \in B \setminus \bigcup_{i=1}^{n} scope(e_i)} (a \leftrightarrow a')
\end{aligned}
$$

# Summary

## Strong planning with full observability

- We have considered the special case of nondeterministic planning where
  - planning tasks are fully observable and
  - we are interested in strong plans.
- We have introduced important concepts also relevant to other variants of nondeterministic planning such as
  - images and
  - weak and strong preimages.
- We have discussed some basic classes of algorithms:
  - forward search in AND/OR graphs, and
  - backward induction by dynamic programming.
- Finally, we have shown how to make a dynamic programming algorithm more efficient by exploiting logic- or set-based representations such as BDDs.