Principles of AI Planning

January 10th, 2007 — Expressive power

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Expressive power

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Motivation: Why Analyzing the Expressive Power?

- Expressive power is the motivation for designing new planning languages
- Often there is the question: *Syntactic sugar* or *essential feature*?

⇝ *Compiling away* or change planning algorithm?

→ If a feature can be compiled away, then it is apparently only *syntactic sugar*.

- Sometimes, however, a compilation can lead to much larger planning domain descriptions or to much longer plans.

⇝ This means the planning algorithm will probably choke, i.e., it cannot be considered as a compilation

Example: DNF Preconditions

- Assume we have **DNF preconditions** in STRIPS operators
- This can be **compiled away** as follows
- Split each operator with a DNF precondition $c_1 \vee \ldots \vee c_n$ into n operators with the same effects and c_i as preconditions

⇝ If there exists a plan for the original planning task there is one for the new planning task and *vice versa*

→ The planning task has almost the same size

→ The shortest plans have the same size

Example: Conditional effects

- Can we compile away **conditional effects** to STRIPS?
- Example operator: $\langle a, b \rhd d \wedge \neg c \rhd e \rangle$
- Can be translated into four operators: $\langle a \wedge b \wedge c, d \rangle, \langle a \wedge b \wedge \neg c, d \wedge e \rangle, \ldots$
- Plan existence and plan size are identical
- Exponential blowup of domain description!

$\longrightarrow$ Can this be avoided?

Propositional STRIPS and Variants

- In the following we will only consider **propositional STRIPS** and some variants of it.
- **Planning task**:

$$\mathcal{T} = \langle A, I, O, G \rangle.$$

- Often we refer to **domain structures** $\mathcal{D} = \langle A, O \rangle$.

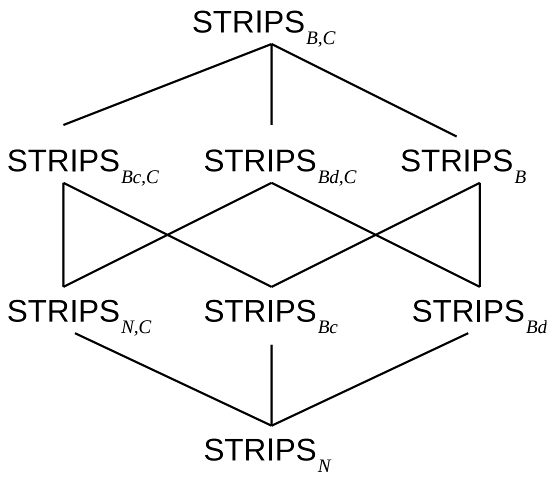
Disjunctive Preconditions: Trivial or Essential?

- Kambhampati et al [ECP 97] and Gazen & Knoblock [ECP 97]: Disjunctive preconditions are trivial – since they can be translated to basic STRIPS (DNF-preconditions)
- Bäckström [AIJ 95]: Disjunctive preconditions are probably essential – since they can not easily be translated to basic STRIPS (CNF-preconditions)
- Anderson et al [AIPS 98]: "[D]isjunctive preconditions . . . are . . . essential prerequisites for handling conditional effects" $\rightsquigarrow$ conditional effects imply disjunctive preconditions (?) (General Boolean preconditions)

More "Expressive Power"

STRIPS_N	:	plain strips with negative literals
STRIPS_{Bd}	:	precondition in disjunctive normal form
STRIPS_{Bc}	:	precondition in conjunctive normal form
STRIPS_B	:	Boolean expressions as preconditions
STRIPS_C	:	conditional effects
$\text{STRIPS}_{C,N}$	:	conditional effects & negative literals

Ordering Planning Formalisms Partially



...

Computational Complexity ...

Theorem

PLANEX is PSPACE-complete for STRIPS_N, $\mathrm{STRIPS}_{C,B}$, and for all formalisms "between" the two.

Proof.

Follows from theorems proved in the previous lecture. □

Measuring Expressive Power

Consider **mappings** between planning problems in different formalisms

- that **preserve**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan "structure"
  - the solutions/plans themselves
- that **are limited**
  - in the *size* of the result (poly. size)
  - in the *computational resources* (poly. time)
- that **transform**
  - entire planning instances
  - domain structure and states in isolation

Method 1: Polynomial Transformation

- **preserving**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan "structure"
  - the solutions/plans themselves
- **limiting**
  - in the *size* of the result (poly. size)
  - in the *computational resources* (poly. time)
- **transforming**
  - entire planning instances
  - domain structure and states in isolation

$\rightsquigarrow$ all formalisms have the **same expressiveness** (?)

Method 2: Bäckström's ESP-reductions

- **preserving**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan "structure"
  - the solutions/plans themselves
- **limiting**
  - in the *size* of the result (poly. size)
  - in the *computational resources* (poly. time)
- **transforming**
  - entire planning instances
  - domain structure and states in isolation

⇝ *However*, **expressiveness** is independent of the **computational resources** needed to compute the mapping

Method 3: Polysize Mappings

- **preserving**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan "structure"
  - the solutions/plans themselves
- **limiting**
  - in the *size* of the result (poly. size)
  - in the *computational resources* (poly. time)
- **transforming**
  - entire planning instances
  - domain structure and states in isolation

⇝ All formalisms are **trivially equivalent** (because planning is PSPACE-complete for all propositional STRIPS formalisms)

Method 4: Modular & Polysize Mappings

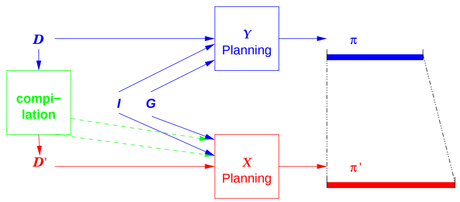
- **preserving**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan "structure"
  - the solutions/plans themselves
- **limiting**
  - in the *size* of the result (poly. size)
  - in the *computational resources* (poly. time)
- **transforming**
  - entire planning instances
  - domain structure and states in isolation

⇝ When measuring the expressiveness of **planning formalisms**, domain structures should be considered independently from states

The Right Method: Compilation Schemes (Simplified)

- Transform **domain structure** $\mathcal{D} = \langle A, O \rangle$ (with polynomial blowup) to $\mathcal{D}'$ preserving solution existence
- Only trivial changes to **states** (independent of operator set)
- Resulting **plans** π' should not grow too much (additive constant, linear growth, polynomial growth)

⇝ Similar to **knowledge compilation**, with operators as the *fixed part* and initial states & goals as the *varying part*



Compilability

$\mathcal{Y} \preceq \mathcal{X}$ ($\mathcal{Y}$ is compilable to $\mathcal{X}$)
iff
there exists a compilation scheme from $\mathcal{Y}$ to $\mathcal{X}$.

$\mathcal{Y} \preceq^1 \mathcal{X}$: preserving plan size **exactly** (modulo additive constants)
$\mathcal{Y} \preceq^c \mathcal{X}$: preserving plan size **linearly** (in $|\pi|$)
$\mathcal{Y} \preceq^p \mathcal{X}$: preserving plan size **polynomially** (in $|\pi|$ and $|\mathcal{D}|$)
$\mathcal{Y} \preceq^x_p \mathcal{X}$: **polynomial-time** compilability

Theorem
For all x, y, the relations $\preceq^x_y$ are transitive and reflexive.

Back-Translatability

- Shouldn't we also require that plans in the compiled instance can be *translated back* to the original formalism?
- Yes, if we want to use this technique, one should require that!
- In all *positive cases*, there was never any problem to translate the plan back
- For the *negative case*, it is easier to prove **non-existence**
- So, in order to prove negative results, we do not need it, for positive it never had been a problem

⇝ So, similarly to the concentration on *decision problems* when determining complexity, we simplify things here

A (Trivial) Positive Result: $\text{STRIPS}_{Bd} \preceq^1_p \text{STRIPS}_N$

DNF preconditions can be **"compiled away."**
Assume operator $o = \langle c, e \rangle$ and

$$c = L_1 \vee \ldots \vee L_k$$

with L_i being a conjunction of literals. Create k operators $o_i = \langle L_i, e \rangle$

1. compilation is solution-preserving,
2. $\mathcal{D}'$ is only polynomially larger than $\mathcal{D}$,
3. compilation can be computed in polynomial time,
4. resulting plans do not grow at all.

⇝ $\text{STRIPS}_{Bd} \preceq^1_p \text{STRIPS}_N$

Another Positive Result: $\text{STRIPS}_{C,Bc} \preceq^c_p \text{STRIPS}_{C,N}$

CNF preconditions can be **"compiled away"** – provided we have already conditional effects.

- Evaluate the truth value of all disjunctions appearing in operators by using a **special evaluation operator** with conditional effects that make new "clause atoms" true
- Alternate between executing original operators (clauses replaced by new atoms) and evaluation operators

⇝ Operator sets grow only **polynomially**

⇝ Plans are **double as long** as the original plans

⇝ **Anderson et al's conjecture** holds in a weak version

A First Negative Result: Conditional Effects Cannot be Compiled into Boolean Preconditions

Consider domain $\mathcal{D}$ with only one ($\text{STRIPS}_{C,B}$) operator o:

$$\langle \top, (p_1 \rhd \neg p_1) \wedge (\neg p_1 \rhd p_1) \wedge \ldots \wedge (p_k \rhd \neg p_k) \wedge (\neg p_k \rhd p_k) \rangle,$$

which "inverts" a given state. For all (I, G) with

$$G = \bigwedge \{ \neg v \mid v \in A, I \models v \} \wedge \bigwedge \{ v \mid v \in A, I \not\models v \},$$

there exists a $\text{STRIPS}_{C,B}$ one-step plan.
Assume there exists a compilation preserving plan size linearly leading to a STRIPS_B domain structure $\mathcal{D}'$. There are exponentially many possible initial states, but only polynomially many different c-step plans for $\mathcal{D}'$. Some STRIPS_B plan π is used for different initial states I_1, I_2 (for large enough k). Let v be a variable with $I_1(v) \neq I_2(v)$.
$\rightsquigarrow$ In one case, v must be set by π, in the other case, it must be cleared.
$\rightsquigarrow$ This is not possible in an unconditional plan.
$\rightsquigarrow$ The transformation is **not solution preserving**!
$\rightsquigarrow$ **Conditional effects** cannot be compiled away (if plan size can grow only linearly)

Another Negative Result: $\text{STRIPS}_{Bc} \not\preceq^c \text{STRIPS}_N$

k-**FISEX**: Planning problem with fixed plan length k and varying initial state. Does there exist an initial state leading to a successful k-step plan?
1-FISEX is NP-complete for STRIPS_{Bc} (= SAT).
k-FISEX is polynomial for STRIPS_N (regression analysis)

$\rightsquigarrow$ $\text{STRIPS}_{Bc} \not\preceq^c_p \text{STRIPS}_N$ (if P≠NP)

Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses **non-uniform complexity classes** such as *P/poly*.

$\rightsquigarrow$ **Bäckström's conjecture holds** in the compilation framework.

A Final Negative Result: Boolean Preconditions Cannot be Compiled Away Even in the Presence of Conditional Effects

- Boolean preconditions have the power of **families of Boolean circuits with logarithmic depth** (because Boolean formula have this power) (= NC^1)
- Conditional effects can simulate only **families of circuits with fixed depth** (= AC^0).
- The parity function can be expressed in the first framework (NC^1) while it cannot be expressed in the second (AC^0).

$\rightsquigarrow$ The negative result follows unconditionally!

Boolean Circuits

- We know what Boolean circuits are (directed, acyclic graphs with different types of nodes: *and, or, not, input, output*)
- **Size of circuit** = number of gates
- **Depth of circuit** = length of longest path from input gate to output gate
- When we want to *recognize formal languages* with circuits, we need a *sequence of circuits* with an increasing number of input gates $\rightsquigarrow$ **family of circuits**
- Families with polynomial size and poly-log ($\log^k n$) depth
- complexity classes NC^k (Nick's class)
- $\text{NC} = \bigcup_k \text{NC}^k \subseteq P$, the class of problems that can be solved efficiently in parallel
- The class of languages that can be characterized by polynomially sized Boolean formulae is identical to NC^1

The classes AC^k

- The classes NC^k are defined with a fixed fan-in
- If we have *unbounded fan-in*, we get the classes AC^k
  - gate types: NOT, n-ary AND, n-ary OR for all $n \geq 2$
- Obviously: $NC^k \subseteq AC^k$
- Possible to show: $AC^{k-1} \subseteq NC^k$
- The *parity language* is in NC^1, but not in AC^0!

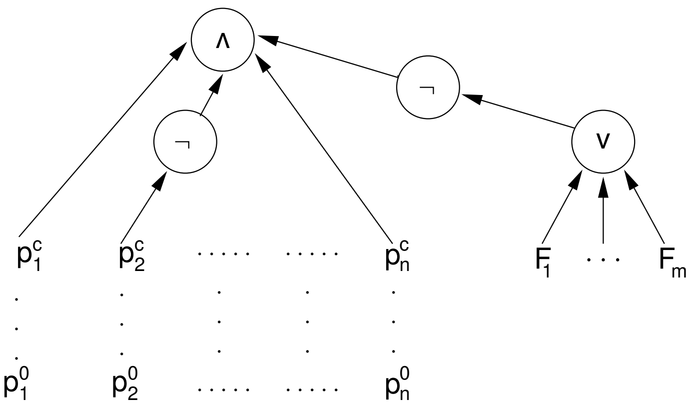
Accepting languages with families of domain structures with fixed goals

- We will view *families of domain structures* with fixed goals and fixed size plans as "machines" that accept languages
- Consider families of poly-sized domain structures in STRIPS_B and use one-step plans for acceptance.
- Obviously, this is the same as using Boolean formulae

$\rightsquigarrow$ All languages in NC^1 can be accepted in this way

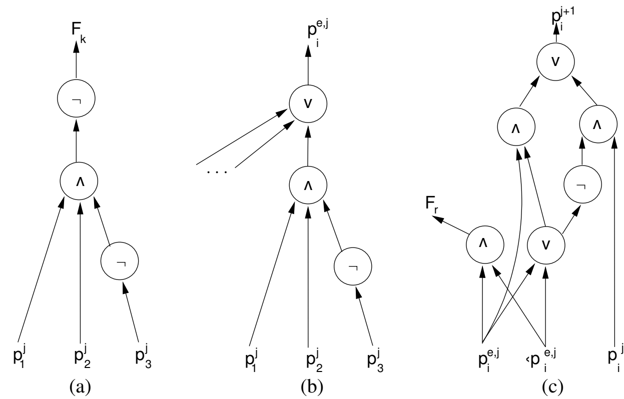
Simulating $\mathrm{STRIPS}_{C,N}$ c-Step Plans with AC^0 circuits (1)

- Represent each operator and then chain the actions together ($O(|O|^c)$ different plans):



Simulating $\mathrm{STRIPS}_{C,N}$ c-Step Plans with AC^0 circuits (2)

- For each single action (precondition testing (a), conditional effects (b), and the computation of effects (c)



$\mathrm{STRIPS}_B \not\preceq^c \mathrm{STRIPS}_{C,N}$

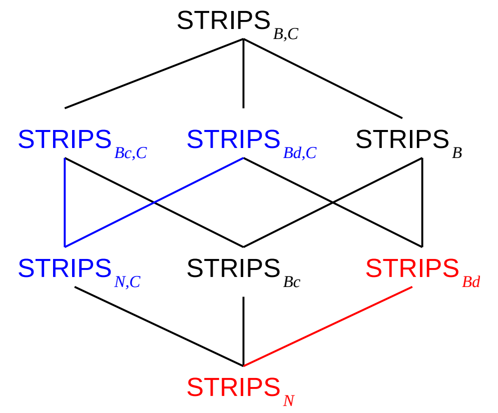
Theorem

$STRIPS_B \not\preceq^c STRIPS_{C,N}$.

Proof.

Assuming $\mathrm{STRIPS}_B \preceq^c \mathrm{STRIPS}_{C,N}$ has the consequence that the underlying compilation scheme could be used to compile a NC^1 circuit family into an AC^0 circuit family, which is impossible in the general case. □

General Results for Compilability Preserving Plan Size Linearly



All other potential positive results have been ruled out by our 3 negative results and transitivity.

Summary

- Compilation schemes seem to be the right method to measure the *relative* expressive power of planning formalisms
- Either we get a positive result preserving plan size **linearly** with a **polynomial-time compilation**
- or we get an **impossibility result**

$\longrightarrow$ **Results are relevant for building planning systems**

$\leadsto$ **CNF preconditions** do not add much when we have already conditional effects

- Note: In all cases we can get a positive result if we allow for a polynomial blow-up of the plans.