

Principles of AI Planning

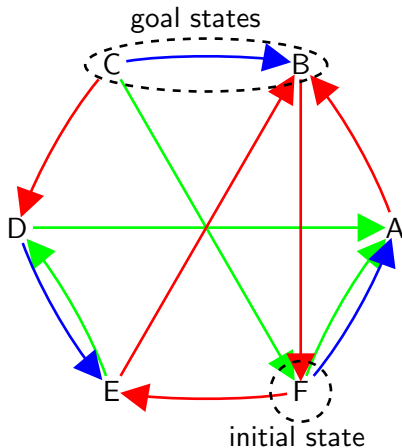
Transition systems

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Transition systems



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Transition systems

Definition

Example

Matrices

Reachability

Algorithm

Succinct TS

Transition systems

Formalization of the dynamics of the world/application

Definition

A **transition system** is $\langle S, I, \{a_1, \dots, a_n\}, G \rangle$ where

- S is a finite set of **states** (the **state space**),
- $I \subseteq S$ is a finite set of **initial states**,
- every action $a_i \subseteq S \times S$ is a binary relation on S ,
- $G \subseteq S$ is a finite set of **goal states**.

Definition

An action a is **applicable** in a state s if sas' for at least one state s' .

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Deterministic transition systems

A transition system is **deterministic** if there is only **one initial state** and all **actions are deterministic**. Hence all future states of the world are completely predictable.

Definition

A **deterministic transition system** is $\langle S, I, O, G \rangle$ where

- S is a finite set of **states** (the **state space**),
- $I \in S$ is a **state**,
- actions $a \in O$ (with $a \subseteq S \times S$) are **partial functions**,
- $G \subseteq S$ is a finite set of **goal states**.

Successor state wrt. an action

Given a state s and an action A so that a is applicable in s , the **successor state** of s with respect to a is s' such that sas' , denoted by $s' = app_a(s)$.

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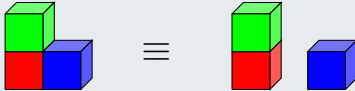
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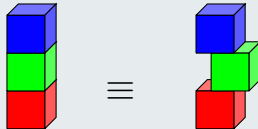
Blocks world

The rules of the game

Location on the table does not matter.



Location on a block does not matter.



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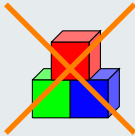
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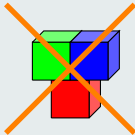
Blocks world

The rules of the game

At most one block may be below a block.



At most one block may be on top of a block.



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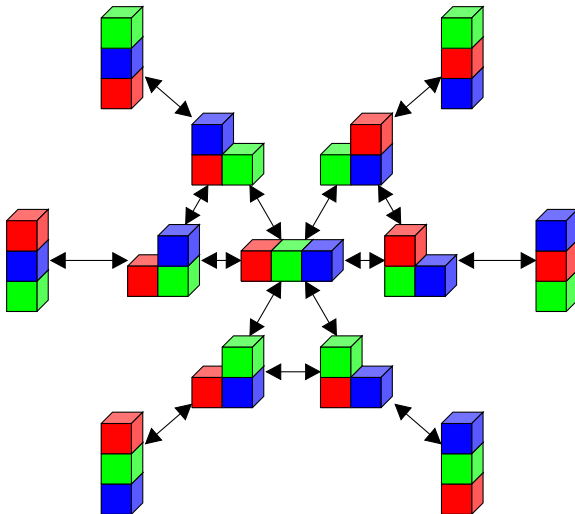
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Blocks world

The transition graph for three blocks



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Blocks world

Properties

| blocks | states |
|--------|----------|
| 1 | 1 |
| 2 | 3 |
| 3 | 13 |
| 4 | 73 |
| 5 | 501 |
| 6 | 4051 |
| 7 | 37633 |
| 8 | 394353 |
| 9 | 4596553 |
| 10 | 58941091 |

- ① Finding a solution is polynomial time in the number of blocks (move everything onto the table and then construct the goal configuration).
- ② Finding a shortest solution is NP-complete (for a compact description of the problem).

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Deterministic planning: plans

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Definition

A **plan** for $\langle S, I, O, G \rangle$ is a sequence $\pi = o_1, \dots, o_n$ of operators such that $o_1, \dots, o_n \in O$ and s_0, \dots, s_n is a sequence of states (the **execution** of π) so that

- ① $s_0 = I$,
- ② $s_i = \text{app}_{o_i}(s_{i-1})$ for every $i \in \{1, \dots, n\}$, and
- ③ $s_n \in G$.

This can be equivalently expressed as

$$\text{app}_{o_n}(\text{app}_{o_{n-1}}(\dots \text{app}_{o_1}(I) \dots)) \in G$$

Transition relations as matrices

- 1 If there are n states, each action (a binary relation) corresponds to an $n \times n$ matrix: The element at row i and column j is 1 if the action maps state i to state j , and 0 otherwise.
For deterministic actions there is at most one non-zero element in each row.
- 2 Matrix multiplication corresponds to **sequential composition**: taking action M_1 followed by action M_2 is the product $M_1 M_2$. (This also corresponds to the **join** of the relations.)
- 3 The unit matrix $I_{n \times n}$ is the NO-OP action: every state is mapped to itself.

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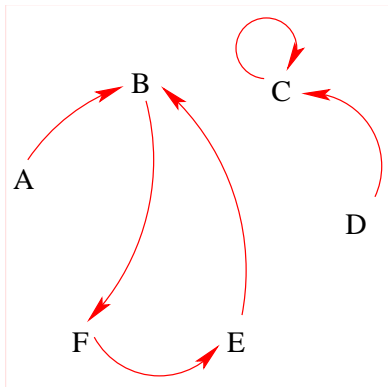
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| | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | 0 | 1 | 0 | 0 | 0 | 0 |
| <i>B</i> | 0 | 0 | 0 | 0 | 0 | 1 |
| <i>C</i> | 0 | 0 | 1 | 0 | 0 | 0 |
| <i>D</i> | 0 | 0 | 1 | 0 | 0 | 0 |
| <i>E</i> | 0 | 1 | 0 | 0 | 0 | 0 |
| <i>F</i> | 0 | 0 | 0 | 0 | 1 | 0 |

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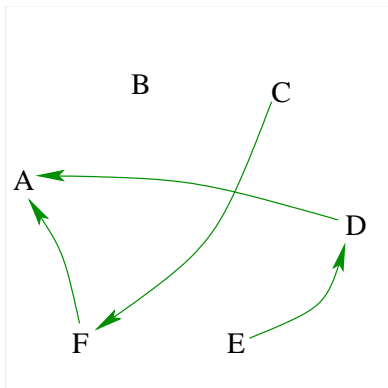
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| | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | 0 | 0 | 0 | 0 | 0 | 0 |
| <i>B</i> | 0 | 0 | 0 | 0 | 0 | 0 |
| <i>C</i> | 0 | 0 | 0 | 0 | 0 | 1 |
| <i>D</i> | 1 | 0 | 0 | 0 | 0 | 0 |
| <i>E</i> | 0 | 0 | 0 | 1 | 0 | 0 |
| <i>F</i> | 1 | 0 | 0 | 0 | 0 | 0 |

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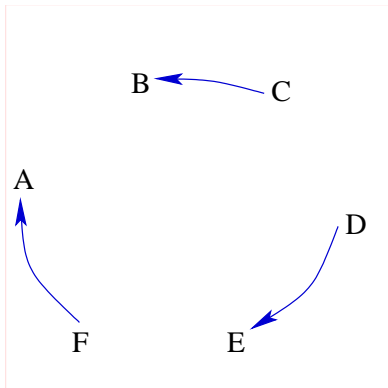
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| | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | 0 | 0 | 0 | 0 | 0 | 0 |
| <i>B</i> | 0 | 0 | 0 | 0 | 0 | 0 |
| <i>C</i> | 0 | 1 | 0 | 0 | 0 | 0 |
| <i>D</i> | 0 | 0 | 0 | 0 | 1 | 0 |
| <i>E</i> | 0 | 0 | 0 | 0 | 0 | 0 |
| <i>F</i> | 1 | 0 | 0 | 0 | 0 | 0 |

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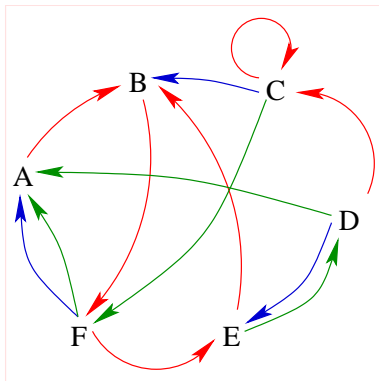
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Sum matrix $M_R + M_G + M_B$

Representing one-step reachability by any of the component actions



| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 0 | 0 | 0 | 1 |
| C | 0 | 1 | 1 | 0 | 0 | 1 |
| D | 1 | 0 | 1 | 0 | 1 | 0 |
| E | 0 | 1 | 0 | 1 | 0 | 0 |
| F | 1 | 0 | 0 | 0 | 1 | 0 |

We use addition $0 + 0 = 0$ and $b + b' = 1$ if $b = 1$ or $b' = 1$.

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Sequential composition as matrix multiplication

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\ \hline 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \times \left(\begin{array}{cccc|c|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & \mathbf{1} & 0 \end{array} \right) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & \mathbf{1} & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

E is reachable from B by two actions
because

F is reachable from B by one action and
E is reachable from F by one action.

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Reachability

Let M be the $n \times n$ matrix that is the (Boolean) sum of the matrices of the individual actions. Define

$$\begin{aligned}R_0 &= I_{n \times n} \\R_1 &= I_{n \times n} + M \\R_2 &= I_{n \times n} + M + M^2 \\R_3 &= I_{n \times n} + M + M^2 + M^3 \\&\vdots\end{aligned}$$

R_i represents reachability by i actions or less. If s' is reachable from s , then it is reachable with $\leq n - 1$ actions: $R_{n-1} = R_n$.

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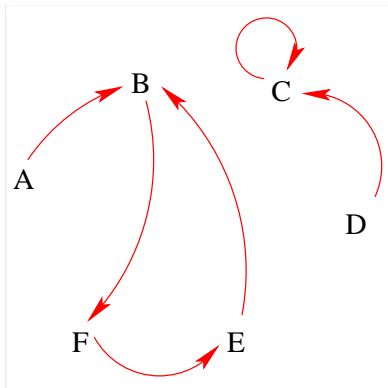
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Reachability: example, M_R



| | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | 0 | 1 | 0 | 0 | 0 | 0 |
| <i>B</i> | 0 | 0 | 0 | 0 | 0 | 1 |
| <i>C</i> | 0 | 0 | 1 | 0 | 0 | 0 |
| <i>D</i> | 0 | 0 | 1 | 0 | 0 | 0 |
| <i>E</i> | 0 | 1 | 0 | 0 | 0 | 0 |
| <i>F</i> | 0 | 0 | 0 | 0 | 1 | 0 |

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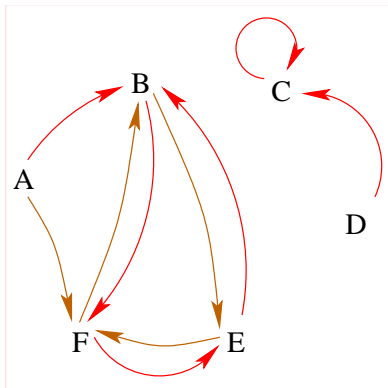
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Reachability: example, $M_R + M_R^2$



| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 0 | 0 | 1 |
| B | 0 | 0 | 0 | 0 | 1 | 1 |
| C | 0 | 0 | 1 | 0 | 0 | 0 |
| D | 0 | 0 | 1 | 0 | 0 | 0 |
| E | 0 | 1 | 0 | 0 | 0 | 1 |
| F | 0 | 1 | 0 | 0 | 1 | 0 |

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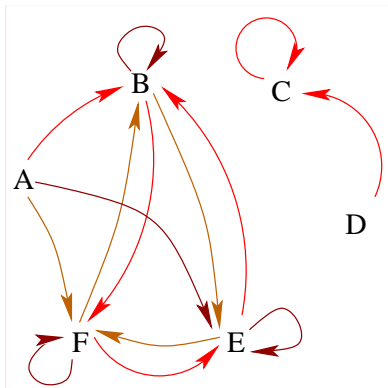
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Reachability: example, $M_R + M_R^2 + M_R^3$



| | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | 0 | 1 | 0 | 0 | 1 | 1 |
| <i>B</i> | 0 | 1 | 0 | 0 | 1 | 1 |
| <i>C</i> | 0 | 0 | 1 | 0 | 0 | 0 |
| <i>D</i> | 0 | 0 | 1 | 0 | 0 | 0 |
| <i>E</i> | 0 | 1 | 0 | 0 | 1 | 1 |
| <i>F</i> | 0 | 1 | 0 | 0 | 1 | 1 |

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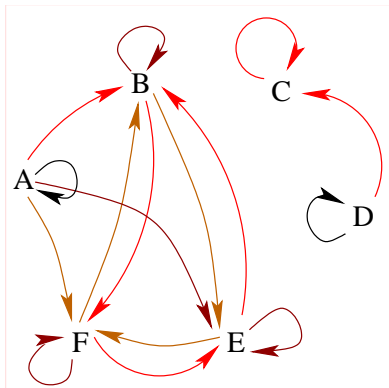
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Reachability: example, $M_R + M_R^2 + M_R^3 + I_{6 \times 6}$



| | <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | 1 | 1 | 0 | 0 | 1 | 1 |
| <i>B</i> | 0 | 1 | 0 | 0 | 1 | 1 |
| <i>C</i> | 0 | 0 | 1 | 0 | 0 | 0 |
| <i>D</i> | 0 | 0 | 1 | 1 | 0 | 0 |
| <i>E</i> | 0 | 1 | 0 | 0 | 1 | 1 |
| <i>F</i> | 0 | 1 | 0 | 0 | 1 | 1 |

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Relations and sets as matrices

Row vectors as sets of states

Row vectors S represent sets.

SM is the set of states reachable from S by M .

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T \times \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}^T$$

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A simple planning algorithm

- We next present a simple planning algorithm based on computing **distances** in the transition graph.
- The algorithm finds shortest paths less efficiently than Dijkstra's algorithm; we present the algorithm because we later will use it as a basis of an algorithm that is applicable to much bigger state spaces than Dijkstra's algorithm directly.

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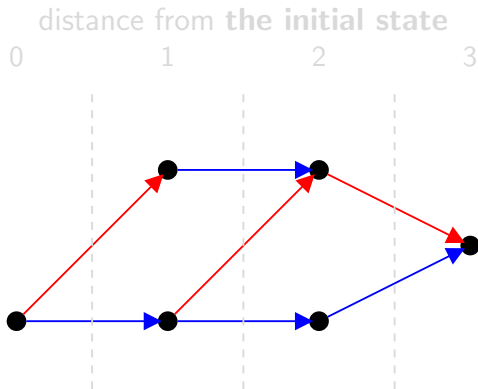
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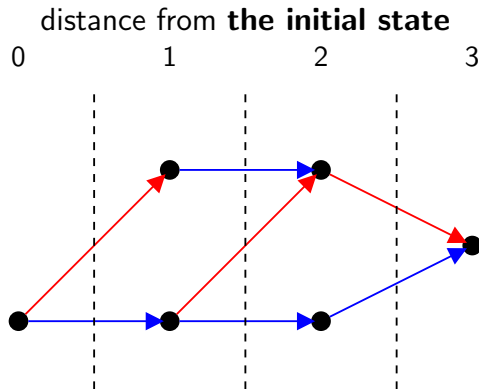
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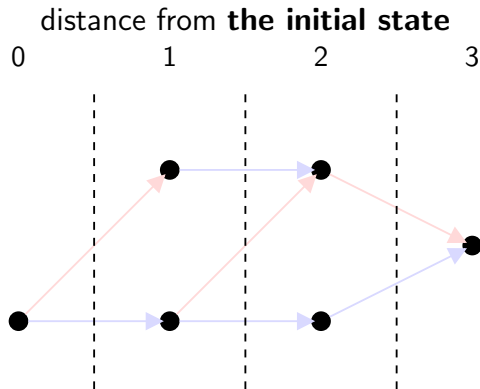
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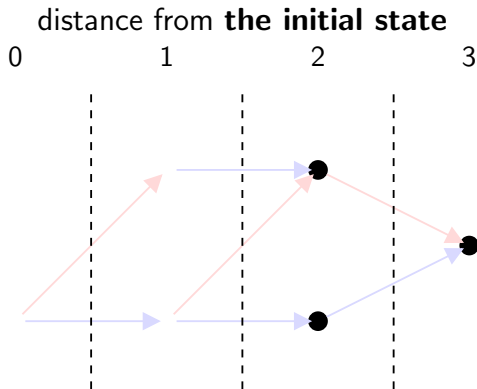
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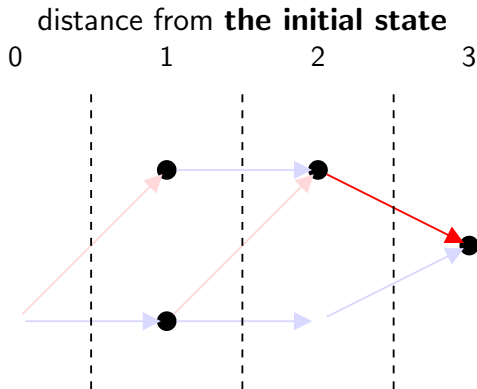
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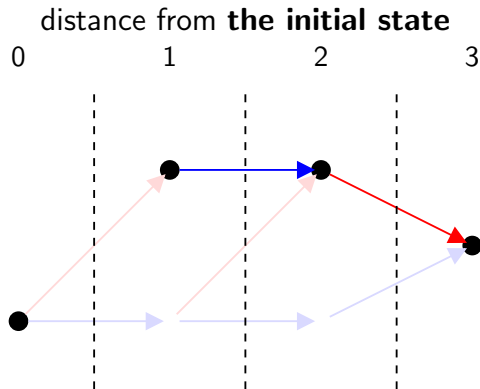
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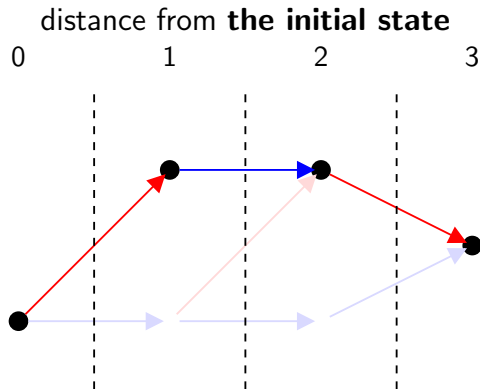
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A simple planning algorithm

- 1 Compute the matrices $R_0, R_1, R_2, \dots, R_n$ representing reachability with $0, 1, 2, \dots, n$ steps with all actions.
- 2 Find the smallest i such that a goal state s_g is reachable from the initial state according to R_i .
- 3 Find an action (the last action of the plan) by which s_g is reached with one step from a state $s_{g'}$ that is reachable from the initial state according to R_{i-1} .
- 4 Repeat the last step, now viewing $s_{g'}$ as the goal state with distance $i - 1$.

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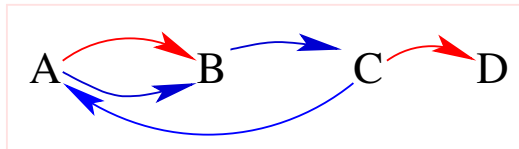
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$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array} + \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & 0 \end{array} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 1 \\ D & 0 & 0 & 0 & 0 \end{array}$$

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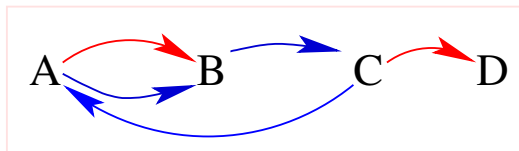
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Example


$$R_0 =$$

| | A | B | C | D |
|---|---|---|---|---|
| A | 1 | 0 | 0 | 0 |
| B | 0 | 1 | 0 | 0 |
| C | 0 | 0 | 1 | 0 |
| D | 0 | 0 | 0 | 1 |

$$R_1 =$$

| | A | B | C | D |
|---|---|---|---|---|
| A | 1 | 1 | 0 | 0 |
| B | 0 | 1 | 1 | 0 |
| C | 1 | 0 | 1 | 1 |
| D | 0 | 0 | 0 | 1 |

$$R_2 =$$

| | A | B | C | D |
|---|---|---|---|---|
| A | 1 | 1 | 1 | 0 |
| B | 1 | 1 | 1 | 1 |
| C | 1 | 1 | 1 | 1 |
| D | 0 | 0 | 0 | 1 |

$$R_3 =$$

| | A | B | C | D |
|---|---|---|---|---|
| A | 1 | 1 | 1 | 1 |
| B | 1 | 1 | 1 | 1 |
| C | 1 | 1 | 1 | 1 |
| D | 0 | 0 | 0 | 1 |

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Succinct representation of transition systems

- More **compact** representation of actions than as relations is often
 - ① **possible** because of symmetries and other regularities,
 - ② **unavoidable** because the relations are too big.
- Represent different aspects of the world in terms of different **state variables**. \implies A state is a **valuation of state variables**.
- Represent actions in terms of changes to the state variables.

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State variables

Logic

Operators

Schemata

State variables

- The state of the world is described in terms of a **finite set** of **finite-valued** state variables.

Example

HOUR : $\{0, \dots, 23\} = 13$

MINUTE : $\{0, \dots, 59\} = 55$

LOCATION : $\{51, 52, 82, 101, 102\} = 101$

WEATHER : $\{\text{sunny, cloudy, rainy}\} = \text{cloudy}$

HOLIDAY : $\{T, F\} = F$

- Any n -valued state variable can be replaced by $\lceil \log_2 n \rceil$ Boolean (2-valued) state variables.
- Actions change the values of the state variables.

Blocks world with state variables

State variables:

$LOCATION_{of}A : \{B, C, TABLE\}$

$LOCATION_{of}B : \{A, C, TABLE\}$

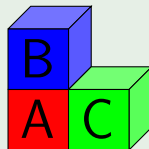
$LOCATION_{of}C : \{A, B, TABLE\}$

Example

$s(LOCATION_{of}A) = TABLE$

$s(LOCATION_{of}B) = A$

$s(LOCATION_{of}C) = TABLE$



Not all valuations correspond to an intended blocks world state, e.g. s such that $s(LOCATION_{of}A) = B$ and $s(LOCATION_{of}B) = A$.

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State variables

Logic

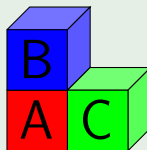
Operators

Schemata

Blocks world with Boolean state variables

Example

| | | |
|--------------------|--------------------|------------------------|
| $s(\text{AonB})=0$ | $s(\text{AonC})=0$ | $s(\text{AonTABLE})=1$ |
| $s(\text{BonA})=1$ | $s(\text{BonC})=0$ | $s(\text{BonTABLE})=0$ |
| $s(\text{ConA})=0$ | $s(\text{ConB})=0$ | $s(\text{ConTABLE})=1$ |



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Logical representations of state sets

- n state variables with m values induce a state space consisting of m^n states (2^n states for n Boolean state variables).
- A language for talking about *sets of states* (*valuations of state variables*) is **propositional logic**.
- Logical connectives correspond to set-theoretical operations.
- Logical relations correspond to set-theoretical relations.

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Propositional logic

Let A be a set of atomic propositions (\sim state variables.)

- ① For all $a \in A$, a is a propositional formula.
- ② If ϕ is a propositional formula, then so is $\neg\phi$.
- ③ If ϕ and ϕ' are propositional formulae, then so is $\phi \vee \phi'$.
- ④ If ϕ and ϕ' are propositional formulae, then so is $\phi \wedge \phi'$.
- ⑤ The symbols \perp and \top are propositional formulae.

The implication $\phi \rightarrow \phi'$ is an abbreviation for $\neg\phi \vee \phi'$.

The equivalence $\phi \leftrightarrow \phi'$ is an abbreviation for
 $(\phi \rightarrow \phi') \wedge (\phi' \rightarrow \phi)$.

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Propositional logic

Valuations and truth

A **valuation** of A is a function $v : A \rightarrow \{0, 1\}$. Define the notation $v \models \phi$ for valuations v and formulae ϕ by

- ① $v \models a$ if and only if $v(a) = 1$, for $a \in A$.
- ② $v \models \neg\phi$ if and only if $v \not\models \phi$
- ③ $v \models \phi \vee \phi'$ if and only if $v \models \phi$ or $v \models \phi'$
- ④ $v \models \phi \wedge \phi'$ if and only if $v \models \phi$ and $v \models \phi'$
- ⑤ $v \models \top$
- ⑥ $v \not\models \perp$

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Propositional logic

Some terminology

- A propositional formula ϕ is **satisfiable** if there is at least one valuation v so that $v \models \phi$. Otherwise it is **unsatisfiable**.
- A propositional formula ϕ is **valid** or a **tautology** if $v \models \phi$ for all valuations v . We write this as $\models \phi$.
- A propositional formula ϕ is a **logical consequence** of a propositional formula ϕ' , written $\phi' \models \phi$, if $v \models \phi$ for all valuations v such that $v \models \phi'$.
- A propositional formula that is a proposition a or a negated proposition $\neg a$ for some $a \in A$ is a **literal**.
- A formula that is a disjunction of literals is a **clause**.

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Formulae vs. sets

| sets | formulae |
|---|--------------------------------------|
| those $\frac{2^n}{2}$ states in which a is true | $a \in A$ |
| $E \cup F$ | $E \vee F$ |
| $E \cap F$ | $E \wedge F$ |
| $E \setminus F$ (set difference) | $E \wedge \neg F$ |
| \overline{E} (complement) | $\neg E$ |
| the empty set \emptyset | \perp |
| the universal set | \top |
| question about sets | question about formulae |
| $E \subseteq F?$ | $E \models F?$ |
| $E \subset F?$ | $E \models F$ and $F \not\models E?$ |
| $E = F?$ | $E \models F$ and $F \models E?$ |

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Operators

Actions are represented as **operators** $\langle c, e \rangle$ where

- c (**the precondition**) is a propositional formula over A describing the set of states in which the action can be taken. (*States in which an arrow starts.*)
- e (**the effect**) describes the successor states of states in which the action can be taken. (*Where do the arrows go.*)
The description is procedural: how do the values of the state variable change?

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Effects

For deterministic operators

Definition

Effects are recursively defined as follows.

- 1 a and $\neg a$ for state variables $a \in A$ are effects.
- 2 $e_1 \wedge \dots \wedge e_n$ is an effect if e_1, \dots, e_n are effects (the special case with $n = 0$ is the empty conjunction \top .)
- 3 $c \triangleright e$ is an effect if c is a formula and e is an effect.

Atomic effects a and $\neg a$ are best understood respectively as assignments $a := 1$ and $a := 0$.

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Effects

Meaning of conditional effects \triangleright

$c \triangleright e$ means that change e takes place if c is true in the current state.

Example

Increment 4-bit numbers $b_3b_2b_1b_0$.

$$\begin{aligned} & (\neg b_0 \triangleright b_0) \wedge \\ & ((\neg b_1 \wedge b_0) \triangleright (b_1 \wedge \neg b_0)) \wedge \\ & ((\neg b_2 \wedge b_1 \wedge b_0) \triangleright (b_2 \wedge \neg b_1 \wedge \neg b_0)) \wedge \\ & ((\neg b_3 \wedge b_2 \wedge b_1 \wedge b_0) \triangleright (b_3 \wedge \neg b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{aligned}$$

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Example: operators for blocks world

In addition to state variables like $AonT$ and $BonC$, for convenience we also use state variables $Aclear$, $Bclear$, and $Cclear$ to denote that there is nothing on the block in question.

$$\langle Aclear \wedge AonT \wedge Bclear, AonB \wedge \neg AonT \wedge \neg Bclear \rangle$$
$$\langle Aclear \wedge AonT \wedge Cclear, AonC \wedge \neg AonT \wedge \neg Cclear \rangle$$
$$\vdots$$
$$\langle Aclear \wedge AonB, AonT \wedge \neg AonB \wedge Bclear \rangle$$
$$\langle Aclear \wedge AonC, AonT \wedge \neg AonC \wedge Cclear \rangle$$
$$\langle Bclear \wedge BonA, BonT \wedge \neg BonA \wedge Aclear \rangle$$
$$\langle Bclear \wedge BonC, BonT \wedge \neg BonC \wedge Cclear \rangle$$
$$\vdots$$

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Operators: meaning

Changes caused by an operator

Assign to each effect e and state s a set $[e]_s$ of literals as follows.

- 1 $[a]_s = \{a\}$ and $[\neg a]_s = \{\neg a\}$ for $a \in A$.
- 2 $[e_1 \wedge \dots \wedge e_n]_s = [e_1]_s \cup \dots \cup [e_n]_s$.
- 3 $[c \triangleright e]_s = [e]_s$ if $s \models c$ and $[c \triangleright e]_s = \emptyset$ otherwise.

Applicability of an operator

Operator $\langle c, e \rangle$ is **applicable in a state s** iff $s \models c$ and $[e]_s$ is consistent.

Operators: the successor state of a state

Definition (Successor state)

The **successor state** $app_o(s)$ of s with respect to operator $o = \langle c, e \rangle$ is obtained from s by making literals in $[e]_s$ true. This is defined only if o is applicable in s .

Example

Consider the operator $\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle$ and a state s such that $s \models a \wedge b \wedge c$.

The operator is applicable because $s \models a$ and $[\neg a \wedge (\neg c \triangleright \neg b)]_s = \{\neg a\}$ is consistent.

Hence $app_{\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle}(s) \models \neg a \wedge b \wedge c$.

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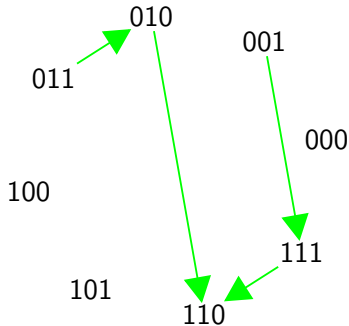
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Example

State variables: $A = \{a, b, c\}$.

An operator is

$$\langle (b \wedge c) \vee (\neg a \wedge b \wedge \neg c) \vee (\neg a \wedge c), \\ ((b \wedge c) \triangleright \neg c) \\ \wedge (\neg b \triangleright (a \wedge b)) \\ \wedge (\neg c \triangleright a) \rangle$$


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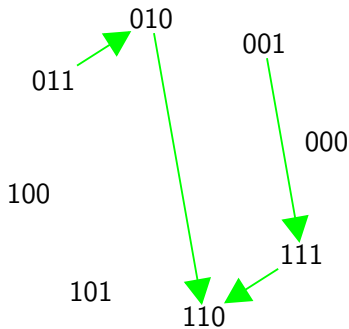
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Example

The corresponding matrix is

| | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 001 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 010 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 011 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 101 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |



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Succinct transition systems

Deterministic case

Definition

A **succinct deterministic transition system** is

$\langle A, I, \{o_1, \dots, o_n\}, G \rangle$ where

- A is a finite set of **state variables**,
- I is an **initial state**,
- every o_i is an operator,
- G is a formula describing the **goal states**.

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Mapping from succinct TS to TS

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From every succinct transition system $\langle A, I, O, G \rangle$ we can produce a corresponding transition system $\langle S, I, O', G' \rangle$.

- 1 S is the set of all valuations of A ,
- 2 $O' = \{R(o) | o \in O\}$ where
 $R(o) = \{(s, s') \in S \times S | s' = \text{app}_o(s)\}$, and
- 3 $G' = \{s \in S | s \models G\}$.

Schematic operators

- Description of state variables and operators in terms of a given finite *set of objects*.
- Analogy: propositional logic vs. predicate logic
- Planners take input as schematic operators, and translate them into (**ground**) operators. This is called **grounding**.

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Schematic operators: example

Schematic operator

$$x \in \{\text{car1}, \text{car2}\}$$

$$y_1 \in \{\text{Freiburg}, \text{Strassburg}\},$$

$$y_2 \in \{\text{Freiburg}, \text{Strassburg}\}, y_1 \neq y_2$$

$$\langle \text{in}(x, y_1), \text{in}(x, y_2) \wedge \neg \text{in}(x, y_1) \rangle$$

corresponds to the operators

$$\begin{aligned} &\langle \text{in}(\text{car1}, \text{Freiburg}), \text{in}(\text{car1}, \text{Strassburg}) \wedge \neg \text{in}(\text{car1}, \text{Freiburg}) \rangle, \\ &\langle \text{in}(\text{car1}, \text{Strassburg}), \text{in}(\text{car1}, \text{Freiburg}) \wedge \neg \text{in}(\text{car1}, \text{Strassburg}) \rangle, \\ &\langle \text{in}(\text{car2}, \text{Freiburg}), \text{in}(\text{car2}, \text{Strassburg}) \wedge \neg \text{in}(\text{car2}, \text{Freiburg}) \rangle, \\ &\langle \text{in}(\text{car2}, \text{Strassburg}), \text{in}(\text{car2}, \text{Freiburg}) \wedge \neg \text{in}(\text{car2}, \text{Strassburg}) \rangle \end{aligned}$$

Schematic operators: quantification

Existential quantification (for formulae only)

Finite disjunctions $\phi(a_1) \vee \dots \vee \phi(a_n)$ represented as $\exists x \in \{a_1, \dots, a_n\} \phi(x)$.

Universal quantification (for formulae and effects)

Finite conjunctions $\phi(a_1) \wedge \dots \wedge \phi(a_n)$ represented as $\forall x \in \{a_1, \dots, a_n\} \phi(x)$.

Example

$\exists x \in \{A, B, C\} \text{in}(x, \text{Freiburg})$ is a short-hand for $\text{in}(A, \text{Freiburg}) \vee \text{in}(B, \text{Freiburg}) \vee \text{in}(C, \text{Freiburg})$.

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PDDL: the Planning Domain Description Language

- Used by almost all implemented systems for deterministic planning.
- Supports a language comparable to what we have defined above (including schematic operators and quantification).
- Syntax inspired by the Lisp programming language:
e.g. prefix notation for formulae.

```
(and (or (on A B) (on A C))  
      (or (on B A) (on B C))  
      (or (on C A) (on A B)))
```

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PDDL: domain files

A domain file consists of

- (define (domain DOMAINNAME))
- a :requirements definition (use :adl :typing by default)
- definitions of types (each parameter has a type)
- definitions of predicates
- definitions of operators

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Example: blocks world in PDDL

```
(define (domain BLOCKS)
  (:requirements :adl :typing)
  (:types block - object
           blueblock smallblock - block)
  (:predicates (on ?x - smallblock ?y - block)
               (ontable ?x - block)
               (clear ?x - block)
               )
)
```

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PDDL: operator definition

- (:action OPERATORNAME
- list of parameters: (?x - type1 ?y - type2 ?z - type3)
- precondition: a formula

`<schematic-state-var>`

`(and <formula> ... <formula>)`

`(or <formula> ... <formula>)`

`(not <formula>)`

`(forall (?x1 - type1 ... ?xn - typen) <formula>)`

`(exists (?x1 - type1 ... ?xn - typen) <formula>)`

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- effect:

```
<schematic-state-var>  
(not <schematic-state-var>)  
(and <effect> ... <effect>)  
(when <formula> <effect>)  
(forall (?x1 - type1 ... ?xn - typen) <effect>)
```

```
(:action fromtable
  :parameters (?x - smallblock ?y - block)
  :precondition (and (not (= ?x ?y))
                     (clear ?x)
                     (ontable ?x)
                     (clear ?y))
  :effect
    (and (not (ontable ?x))
          (not (clear ?y))
          (on ?x ?y)))
```

PDDL: problem files

A problem file consists of

- (define (problem PROBLEMNAME)
- declaration of which domain is needed for this problem
- definitions of objects belonging to each type
- definition of the initial state (list of state variables initially true)
- definition of goal states (a formula like operator precondition)

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```
(define (problem example)
  (:domain BLOCKS)
  (:objects a b c - smallblock
            d e - block
            f - blueblock)
  (:init (clear a) (clear b) (clear c)
         (clear d) (clear e) (clear f)
         (ontable a) (ontable b) (ontable c)
         (ontable d) (ontable e) (ontable f))

  (:goal (and (on a d) (on b e) (on c f)))
)
```

Example run on the FF planner

```
edu/PS04> ./ff -o blocks-dom.pddl -f blocks-ex.pddl
ff: parsing domain file, domain 'BLOCKS' defined
ff: parsing problem file, problem 'EXAMPLE' defined
ff: found legal plan as follows
step      0: FROMTABLE A D
           1: FROMTABLE B E
           2: FROMTABLE C F
0.01 seconds total time
```

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Example: blocks world in PDDL

```
(define (domain BLOCKS)
  (:requirements :adl :typing)
  (:types block)
  (:predicates (on ?x - block ?y - block)
                (ontable ?x - block)
                (clear ?x - block)
                )
)
```

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```
(:action fromtable
  :parameters (?x - block ?y - block)
  :precondition (and (not (= ?x ?y))
                     (clear ?x)
                     (ontable ?x)
                     (clear ?y))
  :effect
    (and (not (ontable ?x))
          (not (clear ?y))
          (on ?x ?y)))
```

```
(:action totable
  :parameters (?x - block ?y - block)
  :precondition (and (clear ?x) (on ?x ?y))
  :effect
    (and (not (on ?x ?y))
          (clear ?y)
          (ontable ?x)))
```

```
(:action move
:parameters (?x - block
             ?y - block
             ?z - block)
:precondition (and (clear ?x) (clear ?z)
                  (on ?x ?y) (not (= ?x ?z)))
:effect
  (and (not (clear ?z))
        (clear ?y)
        (not (on ?x ?y))
        (on ?x ?z)))
```

```
(define (problem blocks-10-0)
  (:domain BLOCKS)
  (:objects d a h g b j e i f c - block)
  (:init (clear c) (clear f)
         (ontable i) (ontable f)
         (on c e) (on e j) (on j b) (on b g)
         (on g h) (on h a) (on a d) (on d i))
  (:goal (and (on d c) (on c f) (on f j)
              (on j e) (on e h) (on h b)
              (on b a) (on a g) (on g i))))
)
```