Motivation

- Conventional NM logics are based on (ad hoc) modifications of the logical machinery (proofs/models).
- Nonmonotonicity is only a negative characterization: If we have $\Theta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \varphi$, we do not necessarily have $\Theta \cup \{\psi\} \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \varphi$.
- Could we have a constructive positive characterization of default reasoning?

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- In conventional logic, we have the logical consequence relation $\alpha \models \beta$: If α is true, then also β is true.
- Instead, we will study the relation of plausible consequence $\alpha \sim \beta$: if α is all we know, can we conclude β ?
- $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta$ does not imply $\alpha \wedge \alpha' \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta!!!$ Compare to conditional probability: $P(\beta|\alpha) \neq P(\beta|\alpha,\alpha')!$
- Find rules characterizing \sim : for example, if $\alpha \sim \beta$ and $\alpha \sim \gamma$, then $\alpha \sim \beta \wedge \gamma$.
- Write down all such rules!

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Desirable Properties 1: Reflexivity

Reflexivity:



- Rationale: If α holds, this *normally implies* α .
- Example: Tom goes to a party normally implies that Tom goes to a party.

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Reflexivity in Default Logic

Plausible consequence as Reasoning in Default Logic

Let us consider relations \sim_{Δ} that are defined in terms of Default Logic.

 $\alpha \mid_{\sim \langle D, W \rangle} \beta$ means that β is a skeptical conclusion of $\langle D, W \cup \{\alpha\} \rangle \mid_{\sim} \beta$.

Proposition

Default Logic satisfies Reflexivity.

Proof

The question is: does α skeptically follow from $\Delta = \langle D, W \cup \{\alpha\} \rangle$?

For all extensions E of Δ , $W \cup \{\alpha\} \subseteq E$ by definition. Hence $\alpha \in E$ and α belongs to all extensions of Δ .

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Desirable Properties 2: Left Logical Equivalence

Left Logical Equivalence:

$$\frac{\models \alpha \leftrightarrow \beta, \ \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

- Rationale: It is not the syntactic form, but the logical content that is responsible for what we conclude normally.
- Example: Assume that
 Tom goes or Peter goes normally implies Mary goes.

Then we would expect that

Peter goes or Tom goes normally implies Mary
goes.

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Left Logical Equivalence in Default Logic

Proposition

Default Logic satisfies Left Logical Equivalence.

Proof.

Assume that $\models \alpha \leftrightarrow \beta$ and γ is in all extensions of $\langle D, W \cup \{\alpha\} \rangle$. The definition of extensions is invariant under replacing any formula by an equivalent formula. Hence $\langle D, W \cup \{\beta\} \rangle$ has exactly the same extensions, and γ is in every one of them.

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Right Weakening:

$$\frac{\models \alpha \to \beta, \ \gamma \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha}{\gamma \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta}$$

- Rationale: If something can be concluded normally, then everything classically implied should also be concluded normally.
- Example: Assume that
 Mary goes normally implies Clive goes and John
 goes.

Then we would expect that
Mary goes *normally implies* Clive goes

• From 1 & 3 supraclassicality follows:

$$\alpha \sim \alpha + \frac{\models \alpha \rightarrow \beta, \, \alpha \mid \sim \alpha}{\alpha \mid \sim \beta} \Rightarrow \frac{\alpha \mid \beta}{\alpha \mid \sim \beta}$$

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$$\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \alpha \hspace{0.2em}+\hspace{0.2em} \begin{array}{c} \hspace{0.2em} \hspace{0.2em} \hspace{0.2em} \hspace{0.2em} \hspace{0.2em} =\hspace{0.2em} \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \alpha \hspace{0.2em}\sim\hspace{0.9em} \alpha \hspace{0.2em}\sim\hspace{0.9em} \beta \hspace{0.2em} \\ \hspace{0.2em} \alpha \hspace{0.2em}\sim\hspace{0.2em} \beta \hspace{0.2em} \end{array} \hspace{0.2em} \Rightarrow \hspace{0.2em} \begin{array}{c} \hspace{0.2em} \alpha \hspace{0.2em} \hspace{0.2em} \hspace{0.2em} \hspace{0.2em} \beta \hspace{0.2em} \\ \hspace{0.2em} \alpha \hspace{0.2em}\sim\hspace{0.9em} \beta \hspace{0.2em} \end{array}$$

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Right Weakening in Default Logic

Proposition

Default Logic satisfies Right Weakening.

Proof.

Assume α is in all extensions of a default theory $\langle D, W \cup \{\gamma\} \rangle$ and $\models \alpha \rightarrow \beta$. Extensions are closed under logical consequence. Hence also β is in all extensions.

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• Cut:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta, \hspace{0.2em} \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

- Rationale: If part of the premise is plausibly implied by another part of the premise, then the latter is enough for the plausible conclusion.
- Example: Assume that
 John goes normally implies Mary goes.

 Assume further that
 John goes and Mary goes normally implied
 - Then we would expect that

 John goes normally implies Clive goes

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• Cut:

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- Example: Assume that
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 Assume further that
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Then we would expect that John goes *normally implies* Clive goes Nonmonotonic Reasoning: Cumulative Logics

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Proposition

Default Logic satisfies Cut.

Proof idea.

Show that every extension E of $\Delta = \langle D, W \cup \{\alpha\} \rangle$ is also an extension of $\Delta' = \langle D, W \cup \{\alpha \land \beta\} \rangle$.

The consistency of the justifications of defaults are tested against E both in the $W \cup \{\alpha\}$ case and in the $W \cup \{\alpha \land \beta\}$ case.

The preconditions that are derivable when starting from $W \cup \{\alpha\}$ are also derivable when starting from $W \cup \{\alpha \land \beta\}$. $W \cup \{\alpha \land \beta\}$ does not allow deriving further preconditions because also with $W \cup \{\alpha\}$ at some point β is derived. Hence E is also an extension of Δ' .

Hence, because γ belongs to all extensions of Δ' , it also belongs to all extensions of Δ .

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Desirable Properties 5: Cautious Monotonicity

Cautious Monotonicity:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta, \hspace{0.2em} \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

- Rationale: In general, adding new premises may cancel some conclusions. However, existing conclusions may be added to the premises without canceling any conclusions!
- Example: Assume that
 Mary goes normally implies Clive goes and
 Mary goes normally implies John goes.
 Mary goes and Jack goes might not normally imply
 that John goes.
 However, Mary goes and Clive goes should
 normally imply that John goes.

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Desirable Properties 5: Cautious Monotonicity

Cautious Monotonicity:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta, \hspace{0.2em} \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

- Rationale: In general, adding new premises may cancel some conclusions. However, existing conclusions may be added to the premises without canceling any conclusions!
- Example: Assume that
 Mary goes normally implies Clive goes and
 Mary goes normally implies John goes.

Mary goes and Jack goes might not normally imply that John goes.

However, Mary goes and Clive goes should normally imply that John goes.

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Cautious Monotonicity in Default Logic

Proposition

Default Logic does not satisfy Cautious Monotonicity.

Proof.

Consider the default theory $\langle D, W \rangle$ with

$$D = \left\{ \frac{a:g}{g}, \frac{g:b}{b}, \frac{b:\neg g}{\neg g} \right\} \text{ and } W = \{a\}.$$

 $E=\operatorname{Th}(\{a,b,g\})$ is the only extension of $\langle D,W\rangle$ and g follows skeptically.

For $\langle D, W \cup \{b\} \rangle$ also Th $(\{a, b, \neg g\})$ is an extension, and g does not follow skeptically.

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Lemma

Rules 4 & 5 can be equivalently stated as follows.

If $\alpha \triangleright \beta$, then the sets of plausible conclusions from α and $\alpha \wedge \beta$ are identical.

The above property is also called cumulativity.

Proof.

 \Rightarrow : Assume that 4 & 5 hold and $\alpha \triangleright \beta$. Assume further that $\alpha \triangleright \gamma$. With rule 5 (CM), we have $\alpha \land \beta \triangleright \gamma$. Similarly, from $\alpha \land \beta \triangleright \gamma$ by rule 4 (Cut) we get $\alpha \triangleright \gamma$.

Hence the plausible conclusions from α and $\alpha \wedge \beta$ are the same.

 \Leftarrow . Assume Cumulativity and $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta$. Now we can derive *rules 4* and 5.

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Reflexivity

$$\overline{\alpha \sim \alpha}$$

2 Left Logical Equivalence

$$\frac{\models \alpha \leftrightarrow \beta, \ \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

Right Weakening

$$\frac{\models \alpha \to \beta, \ \gamma \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha}{\gamma \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta}$$

4 Cut

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta, \hspace{0.2em} \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

Cautious Monotonicity

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• Equivalence:

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MPC:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta \to \gamma, \hspace{0.2em} \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta}{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

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Assumption: $\alpha \sim \beta$, $\alpha \sim \gamma$

Propositional logic: $\alpha \land \beta \land \gamma \models \beta \land \gamma$

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MPC is an Exercise.

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• Monotonicity:

$$\frac{\models \alpha \to \beta, \ \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

- Example: Let us assume that
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 Now we will probably not expect that
 John goes and Joan (who is not in talking terms with
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- Contraposition:

$$\frac{\alpha \sim \beta}{\neg \beta \sim \neg \alpha}$$

• Example: Let us assume that
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Would we expect that
Mary does not go normally implies John does
not go?
What if John goes always?

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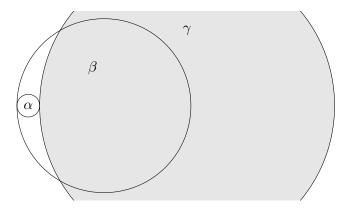
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Undesirable Properties 1: Monotonicity

 $\alpha \models \beta$, $\beta \triangleright \gamma$ but not $\alpha \triangleright \gamma$ pictorially:



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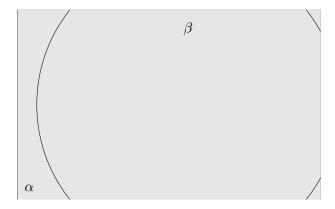
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Undesirable Properties 1: Contraposition

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Transitivity:

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- Example: Let us assume that
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- Easy Half of Deduction Theorem (EHD):

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta \to \gamma}{\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

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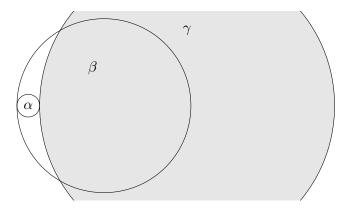
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 $\alpha \triangleright \beta$, $\beta \triangleright \gamma$ but not $\alpha \triangleright \gamma$ pictorially:



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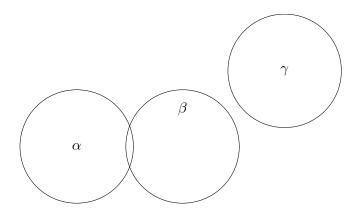
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Undesirable Properties 2: EHD

 $\alpha \triangleright \beta \rightarrow \gamma$ but not $\alpha \land \beta \triangleright \gamma$ pictorially:



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Theorem

In the presence of the rules in system C, monotonicity and EHD are equivalent.

Proof.

Monotonicity \Rightarrow EHD:

- $\alpha \sim \beta \rightarrow \gamma$ (assumption)
- $\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta \to \gamma \hspace{0.2em} \text{(monotonicity)}$
- $\alpha \wedge \beta \sim \alpha \wedge \beta$ (reflexivity)
- $\alpha \wedge \beta \sim \beta$ (right weakening)
- $\alpha \wedge \beta \sim \gamma \text{ (MPC)}$

$Monotonicity \Leftarrow EHD$:

- $\models \alpha \rightarrow \beta, \beta \triangleright \gamma$ (assumption)
- $\beta \sim \alpha \rightarrow \gamma$ (right weakening)
- $\beta \wedge \alpha \sim \gamma$ (EHD)
- $\alpha \sim \gamma$ (left logical equivalence)

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Theorem

In the presence of the rules in system C, monotonicity and transitivity are equivalent.

Proof.

Monotonicity ⇒ *transitivity*:

- $\alpha \sim \beta, \beta \sim \gamma$ (assumption)
- $\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma$ (monotonicity)
- $\alpha \sim \gamma$ (cut)

 $Monotonicity \Leftarrow transitivity$:

- $\models \alpha \rightarrow \beta, \beta \mid \sim \gamma$ (assumption)
- $\alpha \models \beta$ (deduction theorem)
- $\alpha \sim \beta$ (supraclassicality)
- $\alpha \sim \gamma$ (transitivity)

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- $\alpha \sim \gamma$ (transitivity)

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Theorem

In the presence of the rules in system C, monotonicity and transitivity are equivalent.

Proof.

Monotonicity ⇒ *transitivity*.

- $\alpha \sim \beta, \beta \sim \gamma$ (assumption)
- $\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma$ (monotonicity)
- $\alpha \sim \gamma$ (cut)

Monotonicity ← transitivity:

- $\models \alpha \rightarrow \beta, \beta \mid \sim \gamma$ (assumption)
- $\alpha \models \beta$ (deduction theorem)
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Theorem

In the presence of right weakening, contraposition implies monotonicity.

Proof.

- \bigcirc $\models \alpha \rightarrow \beta, \beta \sim \gamma$ (assumption)
- ③ $\models \neg \beta \rightarrow \neg \alpha$ (classical contraposition)

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Theorem

In the presence of right weakening, contraposition *implies* monotonicity.

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- $\bullet \models \alpha \rightarrow \beta, \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma \hspace{0.2em} \text{(assumption)}$
- **3** $\models \neg \beta \rightarrow \neg \alpha$ (classical contraposition)
- $\bigcirc \neg \gamma \sim \neg \alpha$ (right weakening)

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- **3** $\models \neg \beta \rightarrow \neg \alpha$ (classical contraposition)
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Cumulative Closure 1

- How do we *reason* with \sim from φ to ψ ?
- Assumption: We have a set K of conditional statements of the form $\alpha \triangleright \beta$.

 The question is: Assuming the statements in K, is it plausible to conclude ψ given ϕ^2 .
- Idea: We consider all cumulative consequence relations that contain **K**.
- Remark: It suffices to consider only the minimal cumulative consequence relations containing K

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- How do we *reason* with \sim from φ to ψ ?
- Assumption: We have a set K of conditional statements of the form $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta$. The question is: Assuming the statements in K, is it plausible to conclude ψ given φ ?
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- How do we *reason* with \sim from φ to ψ ?
- Assumption: We have a set K of conditional statements of the form $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.5em} \beta.$ The question is: Assuming the statements in K, is it plausible to conclude ψ given φ ?
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Lemma

The set of cumulative consequence relations is closed under intersection.

Proof.

Let $\[\sim_1 \]$ and $\[\sim_2 \]$ be cumulative consequence relations. We have to show that $\[\sim_1 \cap \[\sim_2 \]$ is a cumulative consequence relation, that is, it satisfies the rules 1-5.

Take any instance of the any of the rules. If the preconditions are satisfied by \sim_1 and \sim_2 , then the consequence is trivially also satisfied by both.

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Theorem

For each finite set of conditional statements \mathbf{K} , there exists a unique smallest cumulative consequence relation containing \mathbf{K} .

Proof.

Assume the contrary, i.e., there are incomparable minimal sets $\mathbf{K}_1,\ldots,\mathbf{K}_m$. Then $\mathbf{K}=\mathbf{K}_1\cap\ldots\cap\mathbf{K}_m$ is a unique smallest cumulative consequence relation containing \mathbf{K} : contradiction. This relation is the cumulative closure \mathbf{K}^C of \mathbf{K} .

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- We will now try to characterize cumulative reasoning model-theoretically.
- Idea: Cumulative models consist of states ordered by a preference relation.
- States characterize beliefs.
- The preference relation expresses the normality of the beliefs.
- We say: $\alpha \triangleright \beta$ is accepted in a model if in all most preferred states in which α is true, also β is true.

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Let ≺ be a binary relation on a set U.
 ≺ is asymmetric iff

 $s \prec t$ implies $t \not\prec s$ for all $s, t \in U$.

- Let $V \subseteq U$ and \prec be a binary relation on U.
 - $t \in V$ is minimal in V iff $s \not\prec t$ for all $s \in V$.
 - $t \in V$ is a minimum of V (a smallest element in V) iff $t \prec s$ for all $s \in V$ such that $s \neq t$.
- Let P ⊆ U and ≺ be a binary relation on U.
 P is smooth iff for all t ∈ P, either t is minimal in P or there is s ∈ P such that s is minimal in P and s ≺ t.

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 P is smooth iff for all t ∈ P, either t is minimal in P or there is s ∈ P such that s is minimal in P and s ≺ t.
- Note: ≺ is not a partial order but an arbitrary relation!

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- Let \(\mathcal{U} \) be the set of all possible worlds (propositional interpretations).
- A cumulative model W is a triple $\langle S, l, \prec \rangle$ such that

 - 2 *l* is a mapping $l: S \to 2^{\mathcal{U}}$, and

such that the smoothness condition is satisfied (see below).

- A state $s \in S$ satisfies a formula α ($s \models \alpha$) iff $m \models \alpha$ for all propositional interpretations $m \in l(s)$. The set of states satisfying α is denoted by $\widehat{\alpha}$.
- Smoothness condition: A cumulative model satisfies this condition iff for all formulae α , $\widehat{\alpha}$ is *smooth*.

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such that the smoothness condition is satisfied (see below).

- A state s ∈ S satisfies a formula α (s ⊨ α) iff m ⊨ α for all propositional interpretations m ∈ l(s).
 The set of states satisfying α is denoted by α̂.
- Smoothness condition: A cumulative model satisfies this condition iff for all formulae α , $\widehat{\alpha}$ is *smooth*.

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A cumulative model W induces a consequence relation \sim_W as follows:

 $\alpha \sim_W \beta$ iff $s \models \beta$ for every minimal s in $\widehat{\alpha}$.

Example

Model
$$W = \langle \{s_1, s_2, s_3\}, l, \prec \rangle$$
 with $s_1 \prec s_2, s_2 \prec s_3, s_1 \prec s_3$

$$l(s_1) = \{ \{\neg p, b, f\} \}$$

$$l(s_2) = \{ \{p, b, \neg f\} \}$$

$$l(s_3) = \{ \{\neg p, \neg b, f\}, \{\neg p, \neg b, \neg f\} \}$$

Does W satisfy the smoothness condition?

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$$l(s_2) = \{ \{p, b, \neg f \} \}$$

$$l(s_3) = \{ \{\neg p, \neg b, f \}, \{\neg p, \neg b, \neg f \} \}$$

Does W satisfy the smoothness condition? $\neg p \land \neg b \not\sim f$? N Also: $\neg p \land \neg b \not\sim \neg f$! $p \not\sim \neg f$? Y

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Semantics

A cumulative model W induces a consequence relation \sim_W as follows:

 $\alpha \sim_W \beta$ iff $s \models \beta$ for every minimal s in $\widehat{\alpha}$.

Example

Model
$$W = \langle \{s_1, s_2, s_3\}, l, \prec \rangle$$
 with $s_1 \prec s_2, s_2 \prec s_3, s_1 \prec s_3$

$$l(s_1) = \{ \{\neg p, b, f\} \}$$

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$$\begin{array}{lll}
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p \not \sim \neg f? & \mathsf{Y} \\
\neg p \not \sim f? & \mathsf{Y}
\end{array}$$

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Soundness 1

Theorem

If W is a cumulative model, then \succ_W is a cumulative consequence relation.

Proof

- Reflexivity: satisfied $\sqrt{.}$
- Left logical equivalence: satisfied $\sqrt{.}$
- Right weakening: satisfied $\sqrt{.}$
- Cut: $\alpha \triangleright \beta$, $\alpha \land \beta \triangleright \gamma \Rightarrow \alpha \triangleright \gamma$. Assume that all minimal elements of $\widehat{\alpha}$ satisfy β , and all minimal elements of $\widehat{\alpha} \land \widehat{\beta}$ satisfy γ . Every minimal element of $\widehat{\alpha}$ satisfies $\alpha \land \beta$. Since $\widehat{\alpha} \land \widehat{\beta} \subseteq \widehat{\alpha}$, all minimal elements of $\widehat{\alpha}$ are also minimal elements of $\widehat{\alpha} \land \widehat{\beta}$. Hence $\alpha \triangleright_W \gamma$.

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Proof continues...

• Cautious Monotonicity: To show: $\alpha \triangleright \beta$, $\alpha \triangleright \gamma \Rightarrow \alpha \land \beta \triangleright \gamma$.

Assume $\alpha \hspace{0.2em}\sim_W \beta$ and $\alpha \hspace{0.2em}\sim_W \gamma$. We have to show: $\alpha \wedge \beta \hspace{0.2em}\sim_W \gamma$, i.e., $s \models \gamma$ for all minimal $s \in \widehat{\alpha \wedge \beta}$.

Clearly, every minimal $s \in \widehat{\alpha \wedge \beta}$ is in $\widehat{\alpha}$.

We show that every minimal $s \in \widehat{\alpha} \wedge \widehat{\beta}$ is *minimal* in $\widehat{\alpha}$

Assumption: There is s that is minimal in $\alpha \land \beta$ but not minimal in $\widehat{\alpha}$. Because of smoothness there is minimal $s' \in \widehat{\alpha}$ such that $s' \prec s$. We know, however, that $s' \models \beta$, which means that $s' \in \widehat{\alpha} \land \widehat{\beta}$. Hence s is not minimal in $\widehat{\alpha} \land \widehat{\beta}$. Contradiction! Hence s must be minimal in $\widehat{\alpha}$, and therefore $s \models \gamma$. Because this is true for all minimal elements in $\widehat{\alpha} \land \widehat{\beta}$, we get $\alpha \land \beta \mid_{\sim_W} \gamma$.

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Proof continues...

• Cautious Monotonicity: To show: $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta$, $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma \Rightarrow \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma$. Assume $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \psi \hspace{0.2em} \beta$ and $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \psi \hspace{0.2em} \gamma$. We have to show: $\alpha \wedge \beta \hspace{0.2em}\mid\hspace{0.8em}\mid\hspace{0.8em} \psi \hspace{0.2em} \gamma$, i.e., $s \hspace{0.2em}\mid\hspace{0.8em}\mid\hspace{0.8em} \gamma$ for all minimal $s \in \alpha \wedge \beta$. Clearly, every minimal $s \in \alpha \wedge \beta$ is in $\widehat{\alpha}$.

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Proof continues...

• Cautious Monotonicity: To show: $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta, \, \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma \Rightarrow \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma.$ Assume $\alpha \hspace{0.2em}\mid\hspace{0.8em}\sim\hspace{-0.8em}\mid\hspace{0.8em} \beta$ and $\alpha \hspace{0.2em}\mid\hspace{0.8em}\sim\hspace{-0.8em}\mid\hspace{0.8em} \gamma$. We have to show: $\alpha \wedge \beta \hspace{0.2em}\mid\hspace{0.8em}\sim\hspace{-0.8em}\mid\hspace{0.8em} \gamma$, i.e., $s \hspace{0.2em}\mid\hspace{0.8em}\mid\hspace{0.8em} \gamma$ for all minimal $s \in \widehat{\alpha \wedge \beta}$.

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Proof continues...

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• Cautious Monotonicity: To show: $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta, \, \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma \Rightarrow \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma.$ Assume $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta$ and $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma$. We have to show: $\alpha \wedge \beta \hspace{0.2em}\mid\hspace{0.8em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma$ i.e., $s \hspace{0.2em}\mid\hspace{0.8em} \gamma$ for all minimal $s \in \widehat{\alpha \wedge \beta}$. Clearly, every minimal $s \in \widehat{\alpha \wedge \beta}$ is $\widehat{\alpha}$. We show that every minimal $s \in \widehat{\alpha \wedge \beta}$ is minimal in $\widehat{\alpha}$.

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Proof continues...

• Cautious Monotonicity: To show: $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta, \hspace{0.8em} \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma \Rightarrow \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma.$ Assume $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta$ and $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma$. We have to show: $\alpha \wedge \beta \hspace{0.2em}\mid\hspace{0.8em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma$, i.e., $s \hspace{0.2em}\mid\hspace{0.8em}\mid\hspace{0.8em} \gamma$ for all minimal $s \in \widehat{\alpha \wedge \beta}$. Clearly, every minimal $s \in \widehat{\alpha \wedge \beta}$ is in $\widehat{\alpha}$. We show that every minimal $s \in \widehat{\alpha \wedge \beta}$ is minimal in $\widehat{\alpha}$.

Assumption: There is s that is minimal in $\widehat{\alpha} \wedge \widehat{\beta}$ but not minimal in $\widehat{\alpha}$. Because of *smoothness* there is minimal $s' \in \widehat{\alpha}$ such that $s' \prec s$. We know, however, that $s' \not \equiv \beta$, which means that $s' \in \widehat{\alpha} \wedge \beta$. Hence s is not minimal in $\widehat{\alpha} \wedge \beta$. Contradiction! Hence s must be minimal in $\widehat{\alpha}$, and therefore $s \not \equiv \gamma$. Because this is true for all minimal elements in $\alpha \wedge \beta$, we get $\alpha \wedge \beta \mid_{\neg W} \gamma$.

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Proof continues...

• Cautious Monotonicity: To show: $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta, \ \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma \Rightarrow \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma.$ Assume $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta$ and $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma$. We have to show: $\alpha \wedge \beta \hspace{0.2em}\mid\hspace{0.8em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma$, i.e., $s \hspace{0.2em}\equiv\hspace{0.8em}\mid\hspace{0.8em} \gamma$ for all minimal $s \in \widehat{\alpha \wedge \beta}$. Clearly, every minimal $s \in \widehat{\alpha \wedge \beta}$ is in $\widehat{\alpha}$. We show that every minimal $s \in \widehat{\alpha \wedge \beta}$ is minimal in $\widehat{\alpha}$.

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Proof continues...

• Cautious Monotonicity: To show: $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \beta, \hspace{0.8em} \alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma \Rightarrow \alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em} \gamma.$ Assume $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em}\sim\hspace{-0.9em}\mid\hspace{0.8em}\beta$ and $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em}\sim\hspace{-0.9em}\gamma.$ We have to show: $\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.8em}\sim\hspace{-0.9em}\gamma$, i.e., $s \hspace{0.2em}\equiv\hspace{0.8em}\gamma$ for all minimal $s \in \widehat{\alpha \wedge \beta}$. Clearly, every minimal $s \in \widehat{\alpha \wedge \beta}$ is in $\widehat{\alpha}$. We show that every minimal $s \in \widehat{\alpha \wedge \beta}$ is minimal in $\widehat{\alpha}$.

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Proof continues...

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Proof continues...

- Cautious Monotonicity: To show: $\alpha \triangleright \beta$, $\alpha \triangleright \gamma \Rightarrow \alpha \land \beta \triangleright \gamma$.
 - Assume $\alpha \mid_{\sim_W} \beta$ and $\alpha \mid_{\sim_W} \gamma$. We have to show: $\alpha \land \beta \mid_{\sim_W} \gamma$, i.e., $s \models \gamma$ for all minimal $s \in \widehat{\alpha \land \beta}$.
 - Clearly, every minimal $s \in \widehat{\alpha \wedge \beta}$ is in $\widehat{\alpha}$.
 - We show that every minimal $s \in \widehat{\alpha \wedge \beta}$ is *minimal* in $\widehat{\alpha}$.

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Now we have a method for showing that a principle does not hold for cumulative consequence relations. Simply construct a cumulative model that falsifies the principle.

$$W = \langle S, l, \prec \rangle$$

$$S = \{s_1, s_2, s_3, s_4\}, s_i \not\prec s_j \ \forall s_i, s_j \in S$$

$$l(s_1) = \{\{a, b\}\}\}$$

$$l(s_2) = \{\{\neg a, b\}, \{\neg a, \neg b\}\}$$

$$l(s_3) = \{\{b, a\}, \{b, \neg a\}\}$$

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W is a cumulative model with $a \sim_W b$ but $\neg b \not\sim_W \neg a$

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- Each cumulative model W induces a cumulative consequence relation \succ_W .
- Problem: Can we generate all cumulative consequence relations in this way?
- We can! There is a representation theorem: For each cumulative consequence relation, there is a cumulative model and vice versa.
- Advantage: We have a characterization of the cumulative consequence independently from the set of inference rules.

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- Could we strengthen the preference relation to transitive relations without sacrificing anything?
- In such models, the following additional principle called Loop is valid:

$$\frac{\alpha_o \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_1, \alpha_1 \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_2, \ldots, \alpha_k \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_0}{\alpha_0 \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_k}$$

 For the system CL = C + Loop and cumulative models with transitive preference relations, we could prove another representation theorem. Nonmonotonic Reasoning: Cumulative Logics

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- Could we strengthen the preference relation to transitive relations without sacrificing anything?
 No!
- In such models, the following additional principle called Loop is valid:

$$\frac{\alpha_o \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_1, \alpha_1 \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_2, \ldots, \alpha_k \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_0}{\alpha_0 \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \alpha_k}$$

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Or rule:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma, \hspace{0.2em} \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \vee \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

Not true in C. Counterexample:

$$W = \langle S, l, \prec \rangle$$

$$S = \{s_1, s_2, s_3\}, s_i \not\prec s_j \, \forall s_i, s_j \in S$$

$$l(s_1) = \{\{a, b, c\}, \{a, \neg b, c\}\}\}$$

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 $a \not\sim_W c$, $b \not\sim_W c$ but $a \lor b \not\sim_W c$. Note: Or is not valid in DI Nonmonotonic Reasoning: Cumulative Logics

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Or rule:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma, \hspace{0.2em} \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \vee \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

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 $a \sim_W c$, $b \sim_W c$ but $a \vee b \not\sim_W c$. Note: Or is not valid in DL. Nonmonotonic Reasoning: Cumulative Logics

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Or rule:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma, \hspace{0.2em} \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}{\alpha \vee \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

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 $a \hspace{0.2em}\sim_{\hspace{0.9em} W} c, b \hspace{0.2em}\sim_{\hspace{0.9em} W} c \hspace{0.2em} \text{but } a \vee b \hspace{0.2em}\not\sim_{\hspace{0.9em} W} c.$

Note: Or is not valid in DL.

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Or rule:

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 $a \hspace{0.2em}\sim_{\hspace{0.9em} W} c, b \hspace{0.2em}\sim_{\hspace{0.9em} W} c \hspace{0.2em} \text{but } a \vee b \not \sim_{\hspace{0.9em} W} c.$

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System P

- System P contains all rules of C and the Or rule.
- Derived rules in P
 - Hard half of deduction theorem (S):

$$\frac{\alpha \wedge \beta \sim \gamma}{\alpha \sim \beta \rightarrow \gamma}$$

Proof by case analysis (D):

$$\frac{\alpha \wedge \neg \beta \not \sim \gamma, \ \alpha \wedge \beta \not \sim \gamma}{\alpha \not \sim \gamma}$$

 D and Or are equivalent in the presence of the rules in C. Nonmonotonic Reasoning: Cumulative Logics

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System P

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- System P contains all rules of C and the Or rule.
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Definition

A cumulative model $W=\langle S,l, \prec \rangle$ such that \prec is a *strict* partial order (irreflexive and transitive) and |l(s)|=1 for all $s\in S$ is a preferential model.

Theorem (Soundness)

The consequence relation $\[\sim_W \]$ induced by a preferential model is preferential.

Proof.

Since W is cumulative, we only have to verify that Or holds. Note that in preferential models we have $\widehat{\alpha \vee \beta} = \widehat{\alpha} \cup \widehat{\beta}$. Suppose $\alpha \not\sim_W \gamma$ and $\beta \not\sim_W \gamma$. Because of the above equation, each minimal state of $\widehat{\alpha \vee \beta}$ is minimal in $\widehat{\alpha} \cup \widehat{\beta}$. Since γ is satisfied in all minimal states in $\widehat{\alpha} \cup \widehat{\beta}$, γ is also satisfied in all minimal states of $\widehat{\alpha \vee \beta}$. Hence $\alpha \vee \beta \not\sim_W \gamma$.

Nonmonotonic Reasoning: Cumulative Logics

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Probabilistic Semantics

Theorem (Soundness)

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Theorem (Representation)

A consequence relation is preferential iff it is induced by a preferential model.

Proof.

Similar to the one for C.

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Theorem (Representation)

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Proof.

Similar to the one for C.

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Summary of Consequence Relations

System

+ Loop

+ Or

Reflexivity Left Logical Equivalence Right Weakening Cut Cautious Monotonicity

Models

States: sets of worlds Preference relation: arbitrary Models must be smooth

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Summary of Consequence Relations

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Models

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Preference relation: strict partial order

+ Loop

CL

+ Or

Summary of Consequence Relations

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Models

Reflexivity

Left Logical Equivalence

Right Weakening

Cut

Cautious Monotonicity

States: sets of worlds

Preference relation: arbitrary

Models must be smooth

CL

+ Loop

+ Or

Preference relation: strict partial order

States: singletons

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Strengthening the Consequence Relation

 System C and System P do not produce many of the inferences one would hope for:

```
Given K = \{ \text{Bird} \succ \text{Flies} \} one cannot conclude \text{Red} \land \text{Bird} \succ \text{Flies}!!
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- In general, adding information that is irrelevant cancels the plausible conclusions.

 Cumulative and Preferential consequence relations are too nonmonotonic.
- The plausible conclusions have to be strengthened!!

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Strengthening the Consequence Relation

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Nonmonotonic Reasoning: Cumulative Logics

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Strengthening the Consequence Relations

- The rules so far seem to be reasonable and one cannot think of rules of the same form (if we have some plausible implications, other plausible implications should hold) that could be added.
- However, there are other types of rules one might want add.
- Disjunctive Rationality:

$$\frac{\alpha \not \sim \gamma, \beta \not \sim \gamma}{\alpha \lor \beta \not \sim \gamma}$$

Rational Monotonicity:

$$\frac{\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma \hspace{0.2em},\hspace{0.2em} \alpha \hspace{0.2em}\not\sim\hspace{0.58em} \gamma}{\alpha \wedge \beta \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \gamma}$$

 Note: Consequence relations obeying these rules are not closed under intersection, which is a problem. Nonmonotonic Reasoning: Cumulative Logics

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Strengthening the Consequence Relations

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Preferential Reasoning Preferential

Probabilistic

Probabilistic View of Plausible Consequences

- Consider probability distributions P on the set \mathcal{M} of all interpretations $m \in \mathcal{M}$ (worlds) of our language.
- P(m) is the probability of the possible world m.
- Extend this to probability of formulae:

$$P(\alpha) = \sum \{P(m) | m \in \mathcal{M}, m \models \alpha\}$$

Conditional probability is defined in the standard way.

$$P(\beta|\alpha) = \frac{P(\alpha \wedge \beta)}{P(\alpha)}$$

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ϵ -Entailment

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Definition

 $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.5em} \beta \hspace{0.5em} \text{is } \epsilon\text{-entailed} \hspace{0.5em} \text{by a set } K \hspace{0.5em} \text{iff for all } \epsilon>0 \hspace{0.5em} \text{there is } \delta>0 \hspace{0.5em} \text{such that } P(\beta|\alpha)\geq 1-\epsilon \hspace{0.5em} \text{for all probability distributions } P \hspace{0.5em} \text{such that } P(\beta'|\alpha')\geq 1-\delta \hspace{0.5em} \text{for all } \alpha' \hspace{0.2em}\sim\hspace{-0.9em} \beta' \in K.$

One probability distribution P such that $P(f|b) \ge 0.9$, $P(\neg f|p) \ge 0.9$ and $P(b|p) \ge 0.9$ is the following.

					P(f b)		$\frac{0.99}{1.00}$
	p	b	f	P			1.00
w_1	0	0	0	0.00			
w_2	0	0	1	0.00			
w_3	0	1	0	0.00			
w_{4}	0	1	1	0.99	$P(\neg f p)$		
w_{5}	1	0	0	0.00			
w_{6}	1	0	1	0.00			
w_7	1	1	0	0.01			
w_8	1	1	1	0.00	P(b p)		

Nonmonotonic Reasoning: Cumulative Logics

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One probability distribution P such that $P(f|b) \ge 0.9$, $P(\neg f|p) \ge 0.9$ and $P(b|p) \ge 0.9$ is the following.

	$\mid p$	b	f	P	P(f b)	=	$\frac{P(w_4) + P(w_8)}{P(w_3) + P(w_4) + P(w_7) + P(w_8)} =$	$=\frac{0.99}{1.00}$
w_1	0	0	0	0.00				
	0		1	0.00				
w_3	0	1	0	0.00				
w_{4}	0	1	1	0.99	$P(\neg f p)$			
	1			0.00				
	1		1	0.00				
w_7	1	1	0	0.01				
w_{8}	1	1	1	0.00	P(b p)			

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Maximum Entropy

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	p	b	f	P	P(f b)			$=\frac{0.99}{1.00}$
$\overline{w_1}$	0	0	0	0.00				
	0		1	0.00				
	0	1		0.00			$D(\cdots) + D(\cdots)$	0.01
w_4	0	1	1	0.99	$P(\neg f p)$	=	$\frac{P(w_5)+P(w_7)}{P(w_5)+P(w_6)+P(w_7)+P(w_8)}$	$=\frac{0.01}{0.01}$
w_{5}	1		0	0.00			(), (), (),	
w_{6}	1		1	0.00				
w_{7}	1	1	0	0.01				
w_8	1	1	1	0.00	D/11)			

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	$\mid p \mid$	b	f	P	P(f b)			$=\frac{0.99}{1.00}$
	p	U	J					
w_1	0							
	0		1					
	0	1						
w_4	0	1	1	0.99	$P(\neg f p)$			
w_{5}	1	0		0.00				
w_{6}	1	0	1	0.00				
w_7	1	1		0.01				
w_8	1	1	1	0.00	P(b p)	=	$\frac{P(w_7) + P(w_8)}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} =$	$=\frac{0.01}{0.01}$

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	ı	,	e	l 5	P(f b) =	$\frac{P(w_4) + P(w_8)}{P(w_3) + P(w_4) + P(w_7) + P(w_8)} = \frac{0.99}{1.00}$
	p	b	f	P		
w_1	0	0	0	0.00		
w_2	0	0	1	0.00		
w_3	0	1	0	0.00	- 4 - 1 >	$P(w_5)+P(w_7)$ _ 0.01
w_{4}	0	1	1	0.99	$P(\neg f p) =$	$\frac{P(w_5) + P(w_7)}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} = \frac{0.01}{0.01}$
w_{5}	1	0	0	0.00		
w_{6}	1	0	1	0.00		
w_{7}	1	1	0	0.01		
w_8	1	1	1	0.00	P(b p) =	$\frac{P(w_7) + P(w_8)}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} = \frac{0.01}{0.01}$

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Properties of ϵ -Entailment

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Theorem

 $\alpha \triangleright \beta$ is in all preferential consequence relations that include K if and only if $\alpha \triangleright \beta$ is ϵ -entailed by K.

So, System P provides a proof system that exactly corresponds to ϵ -entailment.

 ϵ -Semantics

- Question: Why is Eagle $\[\sim \]$ Flies not ϵ -entailed by $K = \{ \text{Eagle } \[\sim \]$ Bird, Bird $\[\sim \]$ Flies $\}$?
- Answer: Because there are probability distributions that simultaneously assign very high probabilities to P(Bird|Eagle) and P(Flies|Bird) and a low probability to P(Flies|Eagle).
- K does not justify the low probability of P(Flies|Eagle): there are exactly as many worlds satisfying Bird ∧ Eagle ∧ Flies and Bird ∧ Eagle ∧ ¬Flies, and the worlds satisfying Bird ∧ Flies have a much higher probability than those satisfying Bird ∧ ¬Flies. Why should the probabilities for eagles be the other way round?
- We would like to restrict to probability distributions that are not biased toward non-flying eagles without a reason.

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Entropy of a Probability Distribution

Definition

The entropy of a probability distribution P is

$$H(P) = -\sum_{m \in \mathcal{M}} P(m) \log P(m)$$

The probability distribution with the highest entropy is the one that assigns the same probability to every world.

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Definition

 $\alpha \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta$ is ME-entailed by a set K iff for all $\epsilon > 0$ there is $\delta > 0$ such that $P(\beta|\alpha) \geq 1 - \epsilon$ for the distribution P that has the maximum entropy among distributions satisfying $P(\beta'|\alpha') \geq 1 - \delta$ for all $\alpha' \hspace{0.2em}\sim\hspace{-0.9em}\mid\hspace{0.58em} \beta' \in K$.

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The distribution P that has the maximum entropy among distributions such that $P(b|e) \ge 0.9$ and $P(f|b) \ge 0.9$ is the following.

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					P(f b) =	
	e	b	f	P		
$\overline{w_1}$	0	0	0	0.1875	-	
w_2	0	0	1	0.1875		
w_3	0	1	0	0.0292		
w_{4}	0	1	1	0.1875	P(b e) =	
w_{5}	1	0	0	0.0204		
w_{6}	1	0	1	0.0204		
w_7	1	1	0	0.0292		
w_8	1	1	1	0.3380	$P(f _{\theta}) = 1$	
				-		

The distribution P that has the maximum entropy among distributions such that $P(b|e) \ge 0.9$ and $P(f|b) \ge 0.9$ is the following.

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	$\mid e \mid$	b	f	P	$P(f b) = \frac{P(w_4) + P(w_8)}{P(w_3) + P(w_4) + P(w_7) + P(w_8)} = \frac{0.5255}{0.5839}$
w_1	0	0	0	0.1875	-
	0		1	0.1875	
w_3	0	1	0	0.0292	
w_{4}	0	1	1	0.1875	$P(b e) = \frac{P(w_7) + P(w_8)}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} = \frac{0.3672}{0.4080}$
	1			0.0204	
	1		1	0.0204	
w_7	1	1	0	0.0292	
w_{8}	1	1	1	0.3380	$P(f e) = \frac{P(w_6) + P(w_8)}{P(w_8) + P(w_8) + P(w_8) + P(w_8)} = \frac{0.3584}{0.4080} =$

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Nonmonotonic Reasoning: Cumulative Logics

Maximum Entropy

	$\mid e \mid$	b	f	$P(f b) = \frac{P(w_4) + P(w_8)}{P(w_3) + P(w_4) + P(w_7) + P(w_8)} = \frac{0.5255}{0.5839}$
w_1	0	0	0	0.1875
	0		1	0.1875
	0	1		0.0292
w_4	0	1	1	$0.1875 P(b e) = \frac{P(w_7) + P(w_8)}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} = \frac{0.3672}{0.4080}$
w_{5}	1	0		0.0204
w_{6}	1	0	1	0.0204
w_{7}	1	1		0.0292
w_8	1	1	1	$P(f e) = \frac{P(w_6) + P(w_8)}{P(w_8) + P(w_8) + P(w_7) + P(w_8)} = \frac{0.3584}{0.4080} = \frac{0.3584}{0.4080}$

The distribution P that has the maximum entropy among distributions such that $P(b|e) \ge 0.9$ and $P(f|b) \ge 0.9$ is the following.

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Maximum Entropy

Literature

	$\mid e \mid$	b	f	$P(f b) = \frac{P(w_4) + P(w_8)}{P(w_3) + P(w_4) + P(w_7) + P(w_8)} = \frac{0.5255}{0.5839}$
$\overline{w_1}$	0	0	0	0.1875
	0		1	0.1875
	0	1		0.0292
w_4	0	1	1	0.1875 $P(b e) = \frac{P(w_7) + P(w_8)}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} = \frac{0.3672}{0.4080}$
w_5	1		0	0.0204
w_{6}	1		1	0.0204
w_7	1	1	0	0.0292
w_{8}	1	1	1	$ P(f e) = \frac{P(w_6) + P(w_8)}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} = \frac{0.3584}{0.4080} = $
				$P(y) = \frac{1}{P(w_5) + P(w_6) + P(w_7) + P(w_8)} = \frac{1}{0.4080} = \frac{1}{0.4080}$

The distribution P that has the maximum entropy among distributions such that $P(b|e) \ge 0.9$ and $P(f|b) \ge 0.9$ is the following.

Nonmonotonic Reasoning: Cumulative Logics

System C

easoning

Preferential

Reasoning

Semantics ←-Semantics

Maximum Entropy

- ② {Penguin | Bird, Bird | Flies, Penguin | ¬Flies}
- ◆ {Bat | Mammal, Bat | Winged-Animal,

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Semantics ε-Semantics Maximum Entropy

Maximum Entrop

- ◆ {Eagle ~ Bird, Bird ~ Flies} ME-entails Eagle ~ Flies

- § Bat

 Mammal, Bat

 Winged-Animal,

 Mammal,

 Mamma Winged-Animal \triangleright Flies, Mammal $\triangleright \neg$ Flies} does not ME-entail Bat \sim Flies.

Nonmonotonic Reasoning Cumulative Logics

Maximum Entropy

- {Bat ├ Mammal, Bat ├ Winged-Animal, Winged-Animal ├ Flies, Mammal ├ ¬Flies} does not ME-entail Bat ├ Flies.

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Maximum Entropy

ME-Entailment: Problems

- No good proof systems and reasoning algorithms exist!
 Computing maximum entropy distributions is computationally extremely complex when the number of worlds is high.
- The way knowledge is expressed still has a strong impact on what conclusions can be derived.

does not ME-entail anything about flying ability of bats.

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```
 \begin{array}{ll} \text{Bat} \hspace{0.2em} |\hspace{-0.2em} \hspace{0.2em} \text{Mammal}, & \text{Mammal} \hspace{0.2em} |\hspace{-0.2em} \hspace{0.2em} \neg \text{Winged-Animal}, \\ \text{Bat} \hspace{0.2em} |\hspace{-0.2em} \hspace{0.2em} \text{Winged-Animal} \hspace{0.2em} |\hspace{-0.2em} \hspace{0.2em} \neg \text{Flies} \\ \text{Mammal} \hspace{0.2em} |\hspace{-0.2em} \hspace{0.2em} \neg \text{Flies} \} \\ \end{array}
```

ME-entails Bat \vdash Flies.

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Probabilistic Semantics

Maximum Entropy Literature

Summary

- Instead of ad hoc extensions of the logical machinery, analyze the properties of nonmonotonic consequence relations.
- Correspondence between rule system and models for System C, and for System P also wrt a probabilistic semantics.
- Irrelevant information poses a problem. Solution approaches: rational monotony, maximum entropy, ...

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Sarit Kraus, Daniel Lehmann, and Menachem Magidor. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence*, 44:167–207, 1990. Introduces Cumulative

consequence relations.

Daniel Lehman and Menachem Magidor. What does a conditional knowledge base entail? *Artificial Intelligence*, 55:1–60, 1992.

Introduces rational consequence relation

Dov M. Gabbay. Theoretical foundations for non-monotonic reasoning in expert systems. In K. R. Apt, editor, *Proceedings NATO Advanced Study Institute on Logics and Models of Concurrent Systems*, pages 439–457. Springer-Verlag, Berlin, Heidelberg, New York, 1985.

First to consider abstract properties of nonmonotonic consequence relations.

Judea Pearl. *Probabilistic Reasoning in Intelligent Systems:*Networks of Plausible Inference, Morgan Kaufmann Publishers,
1988. One section on e-semantics and maximum entropy.

Yoav Shoham. *Reasoning about Change*. MIT Press, Cambridge, MA 1988. Introduces the idea of preferential models.

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