- In propositional logic, the only building blocks are atomic propositions.
- We cannot talk about the internal structures of these propositions.
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  - All CS students know formal logic
  - Peter is a CS student
  - Therefore, Peter knows formal logic
  - Not possible in propositional logic
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Classical Logic

First Order Logic

Motivation

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Semantics

Litaratura

- variable symbols: x, y, z, ...
- *n*-ary function symbols:  $f(\cdots), g(\cdots), \ldots$
- constant symbols:  $a, b, c, \ldots$
- *n*-ary predicate symbols:  $P(\cdots), Q(\cdots), \ldots$

Terms

$$egin{array}{lll} t & \longrightarrow x & ext{variable} \ & & f(t_1,\dots,t_n) ext{ function application} \ & & a & ext{constant} \end{array}$$

Formulae

$$\longrightarrow P(t_1,\ldots,t_n)$$
 atomic formula propositional connectives  $\forall x(\varphi')$  universal quantification  $\exists x(\varphi')$  existential quantification

ground term, etc.: term, etc. without variable occurrences

Classical Logic

First Order Logic

Syntax

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Terminology

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Classical Logic

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Syntax

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Classical Logic

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**Terms** 

 $t \longrightarrow x$  variable

 $f(t_1, \dots, t_n)$  function application a constant

Formulae

 $\longrightarrow P(t_1,\ldots,t_n)$  atomic formula propositional connections

 $\forall x(\varphi')$  universal quantification  $\exists x(\varphi')$  existential quantification

ground term, etc.: term, etc. without variable occurrences

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Formulae

 $\neg F(t_1, \dots, t_n)$  atomic formula  $\neg F(t_1, \dots, t_n)$  atomic formula  $\neg F(t_1, \dots, t_n)$  propositional connectives  $\neg F(t_1, \dots, t_n)$  universal quantification  $\neg F(t_1, \dots, t_n)$  existential quantification

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Formulae

$$arphi \longrightarrow P(t_1,\dots,t_n)$$
 atomic formula  $| \dots |$  propositional connectives  $| \forall x(arphi') |$  universal quantification  $| \exists x(arphi') |$  existential quantification

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#### Semantics: Idea

- In FOL, the universe of discourse consists of objects, functions over these objects, and relations over these objects.
- Function symbols are mapped to functions, predicate symbols are mapped to relations, and terms to objects.
- Notation: Instead of  $\mathcal{I}(x)$  we write  $x^{\mathcal{I}}$ .
- Note: Usually one considers all possible non-empty universes. (However, sometimes the interpretations are restricted to particular domains, e.g. integers or real numbers.)
- Satisfiability and validity is then considered wrt all these universes.

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Interpretations:  $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$  with  $\mathcal{D}$  being an arbitrary non-empty set and  $\mathcal{I}$  being a function which maps

- n-ary function symbols f to n-ary functions  $f^{\mathcal{I}} \in [\mathcal{D}^n \to \mathcal{D}],$
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$$(f(t_1,\ldots,t_n))^{\mathcal{I}} = f^{\mathcal{I}}(t_1^{\mathcal{I}},\ldots,t_n^{\mathcal{I}}) \in \mathcal{D}$$

Truth of ground atoms:

$$\mathcal{I} \models P(t_1, \dots, t_n) \text{ iff } \langle {t_1}^{\mathcal{I}}, \dots, {t_n}^{\mathcal{I}} \rangle \in P^{\mathcal{I}}$$

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# $\mathcal{D} = \{d_1, \dots, d_n\}, n \geq 2 \quad \mathcal{D} = \{1, 2, 3, \dots\}$ $\mathbf{a}^{\mathcal{I}} = d_1 \qquad \mathbf{1}^{\mathcal{I}} = 1$ $\mathbf{b}^{\mathcal{I}} = d_2 \qquad \mathbf{2}^{\mathcal{I}} = 2$ $\mathbf{eye}^{\mathcal{I}} = \{d_1\} \qquad \vdots$ $\mathbf{red}^{\mathcal{I}} = \mathcal{D} \qquad \mathbf{even}^{\mathcal{I}} = \{2, 4, 6, \dots\}$ $\mathcal{I} \models \mathbf{red}(\mathbf{b}) \qquad \mathbf{succ}^{\mathcal{I}} = \{(1 \mapsto 2), (2 \mapsto 3), \dots\}$ $\mathcal{I} \not\models \mathbf{eye}(\mathbf{b}) \qquad \mathcal{I} \not\models \mathbf{even}(\mathbf{3})$ $\mathcal{I} \models \mathbf{even}(\mathbf{succ}(\mathbf{3}))$

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#### Classical Logic

Interpretations

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V is the set of variables. Function  $\alpha \colon V \to \mathcal{D}$  is a variable map.

Notation:  $\alpha[x/d]$  is identical to  $\alpha$  except for x where  $\alpha[x/d](x) = d$ .

Interpretation of terms under  $\mathcal{I}, \alpha$ :

$$x^{\mathcal{I},\alpha} = \alpha(x)$$

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Example (cont.):

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Truth of  $\varphi$  by  $\mathcal{I}$  under  $\alpha$  ( $\mathcal{I}, \alpha \models \varphi$ ) is defined as follows.

$$\mathcal{I}, \alpha \models P(t_1, \dots, t_n) \quad \text{iff} \quad \langle t_1^{\mathcal{I}, \alpha}, \dots, t_n^{\mathcal{I}, \alpha} \rangle \in P^{\mathcal{I}}$$

$$\mathcal{I}, \alpha \models \neg \varphi \quad \text{iff} \quad \mathcal{I}, \alpha \not\models \varphi$$

$$\mathcal{I}, \alpha \models \varphi \land \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ and } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \varphi \lor \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ or } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \varphi \to \psi \quad \text{iff} \quad \text{if } \mathcal{I}, \alpha \models \varphi, \text{ then } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \varphi \to \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi, \text{ iff } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \forall x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for all } d \in \mathcal{D}$$

$$\mathcal{I}, \alpha \models \exists x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for some } d \in \mathcal{D}$$

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Truth of  $\varphi$  by  $\mathcal{I}$  under  $\alpha$  ( $\mathcal{I}, \alpha \models \varphi$ ) is defined as follows.

$$\mathcal{I}, \alpha \models P(t_1, \dots, t_n) \quad \text{iff} \quad \langle {t_1}^{\mathcal{I}, \alpha}, \dots, {t_n}^{\mathcal{I}, \alpha} \rangle \in P^{\mathcal{I}}$$

$$\mathcal{I}, \alpha \models \neg \varphi \quad \text{iff} \quad \mathcal{I}, \alpha \not\models \varphi$$

$$\mathcal{I}, \alpha \models \varphi \land \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ and } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \varphi \lor \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ or } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \varphi \to \psi \quad \text{iff} \quad \text{if } \mathcal{I}, \alpha \models \varphi, \text{ then } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \varphi \leftrightarrow \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi, \text{ iff } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \forall x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for all } d \in \mathcal{D}$$

$$\mathcal{I}, \alpha \models \exists x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for some } d \in \mathcal{D}$$

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Definition of truth

Truth of  $\varphi$  by  $\mathcal{I}$  under  $\alpha$  ( $\mathcal{I}, \alpha \models \varphi$ ) is defined as follows.

$$\mathcal{I}, \alpha \models P(t_1, \dots, t_n) \quad \text{iff} \quad \langle t_1^{\mathcal{I}, \alpha}, \dots, t_n^{\mathcal{I}, \alpha} \rangle \in P^{\mathcal{I}}$$

$$\mathcal{I}, \alpha \models \neg \varphi \quad \text{iff} \quad \mathcal{I}, \alpha \not\models \varphi$$

$$\mathcal{I}, \alpha \models \varphi \land \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ and } \mathcal{I}, \alpha \models \psi$$

$$\mathcal{I}, \alpha \models \varphi \lor \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ or } \mathcal{I}, \alpha \models \psi$$

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$$\mathcal{I}, \alpha \models \forall x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for all } d \in \mathcal{D}$$

$$\mathcal{I}, \alpha \models \exists x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for some } d \in \mathcal{D}$$

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Truth of  $\varphi$  by  $\mathcal{I}$  under  $\alpha$  ( $\mathcal{I}, \alpha \models \varphi$ ) is defined as follows.

$$\begin{split} \mathcal{I}, \alpha &\models P(t_1, \dots, t_n) \quad \text{iff} \quad \langle {t_1}^{\mathcal{I}, \alpha}, \dots, {t_n}^{\mathcal{I}, \alpha} \rangle \in P^{\mathcal{I}} \\ \mathcal{I}, \alpha &\models \neg \varphi \quad \text{iff} \quad \mathcal{I}, \alpha \not\models \varphi \\ \mathcal{I}, \alpha &\models \varphi \land \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ and } \mathcal{I}, \alpha \models \psi \\ \mathcal{I}, \alpha &\models \varphi \lor \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi \text{ or } \mathcal{I}, \alpha \models \psi \\ \mathcal{I}, \alpha &\models \varphi \to \psi \quad \text{iff} \quad \text{if } \mathcal{I}, \alpha \models \varphi, \text{ then } \mathcal{I}, \alpha \models \psi \\ \mathcal{I}, \alpha &\models \varphi \leftrightarrow \psi \quad \text{iff} \quad \mathcal{I}, \alpha \models \varphi, \text{ iff } \mathcal{I}, \alpha \models \psi \\ \mathcal{I}, \alpha &\models \forall x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for all } d \in \mathcal{D} \\ \mathcal{I}, \alpha &\models \exists x \varphi \quad \text{iff} \quad \mathcal{I}, \alpha[x/d] \models \varphi \text{ for some } d \in \mathcal{D} \end{split}$$

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#### Questions:

$$\begin{array}{lll} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

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#### Questions:

$$\begin{array}{lcl} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

```
\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?
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#### Questions:

$$\begin{array}{lcl} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a),\operatorname{eye}(b) \\ \forall x(\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1,\ldots,d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

$$\mathcal{I}, \alpha \models \mathsf{eye}(b) \lor \neg \mathsf{eye}(b)$$
?

Yes

 $\mathcal{I}, \alpha \models \mathsf{eye}(x) \to \mathsf{eye}(x) \lor \mathsf{eye}(y)$ ? Yes

 $\mathcal{I}, \alpha \models \mathsf{eye}(x) \to \mathsf{eye}(y)$ ? No

 $\mathcal{I}, \alpha \models \mathsf{eye}(a) \land \mathsf{eye}(b)$ ?

Yes

 $\mathcal{I}, \alpha \models \forall x (\mathsf{eye}(x) \to \mathsf{red}(x))$ ? Yes

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#### Questions:

$$\begin{array}{lll} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

$$\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?$$
 Yes 
$$\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow \\ \mathsf{eye}(x) \vee \mathsf{eye}(y)?$$
 Yes 
$$\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow \\ \mathsf{eye}(y)?$$
 No 
$$\mathcal{I}, \alpha \models \mathsf{eye}(a) \wedge \mathsf{eye}(b)?$$
 Yes 
$$\mathcal{I}, \alpha \models \forall x (\mathsf{eye}(x) \rightarrow \\ \mathsf{red}(x))?$$
 Yes 
$$\mathcal{I}, \alpha \models \forall x (\mathsf{eye}(x) \rightarrow \\ \mathsf{red}(x))?$$

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#### Questions:

$$\begin{array}{lcl} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

```
\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow
eye(x) \lor eye(y)? Yes
```

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#### Questions:

$$\begin{array}{lcl} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

$$\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?$$

$$\mathsf{Yes}$$

$$\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow \\ \mathsf{eye}(x) \vee \mathsf{eye}(y)? \quad \mathsf{Yes}$$

$$\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow \\ \mathsf{eye}(y)? \quad \mathsf{No}$$

$$\mathcal{I}, \alpha \models \mathsf{eye}(a) \wedge \mathsf{eye}(b)?$$

$$\mathsf{Yes}$$

$$\mathcal{I}, \alpha \models \forall x (\mathsf{eye}(x) \rightarrow \\ \mathsf{red}(x))? \quad \mathsf{Yes}$$

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#### Questions:

$$\begin{array}{lll} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

```
\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow
eye(x) \lor eye(y)? Yes
\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow
eye(y)? No
```

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#### Questions:

$$\begin{array}{lcl} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

```
 \begin{array}{l} \mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)? \\ \textcolor{red}{\mathsf{Yes}} \\ \mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow \\ \textcolor{red}{\mathsf{eye}(x)} \vee \textcolor{red}{\mathsf{eye}(y)?} \textcolor{black}{\texttt{Yes}} \\ \mathcal{I}, \alpha \models \textcolor{red}{\mathsf{eye}(x)} \rightarrow \\ \textcolor{red}{\mathsf{eye}(y)?} \textcolor{black}{\texttt{No}} \\ \mathcal{I}, \alpha \models \textcolor{red}{\mathsf{eye}(a)} \wedge \textcolor{red}{\mathsf{eye}(b)?} \\ \textcolor{red}{\mathsf{Yes}} \\ \mathcal{I}, \alpha \models \forall x (\textcolor{red}{\mathsf{eye}(x)} \rightarrow \\ \textcolor{red}{\mathsf{red}(x))?} \textcolor{black}{\texttt{Yes}} \\ \end{array}
```

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### Questions:

$$\begin{array}{l} \mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)? \\ \textcolor{red}{\textit{Yes}} \\ \mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow \\ \mathsf{eye}(x) \vee \mathsf{eye}(y)? \textcolor{red}{\textit{Yes}} \\ \mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow \\ \mathsf{eye}(y)? \textcolor{red}{\textit{No}} \\ \mathcal{I}, \alpha \models \mathsf{eye}(a) \wedge \mathsf{eye}(b)? \\ \textcolor{red}{\textit{Yes}} \\ \mathcal{I}, \alpha \models \forall x (\mathsf{eye}(x) \rightarrow \\ \mathsf{red}(x))? \textcolor{red}{\textit{Yes}} \end{array}$$

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#### Questions:

$$\begin{array}{lcl} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

```
\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \text{eve}(x) \rightarrow
eye(x) \lor eye(y)? Yes
\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow
eye(y)? No
\mathcal{I}, \alpha \models \mathsf{eye}(a) \land \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \forall x (eye(x) \rightarrow
red(x)? Yes
```

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#### Questions:

$$\begin{array}{lll} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

```
\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \text{eve}(x) \rightarrow
eye(x) \lor eye(y)? Yes
\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow
eye(y)? No
\mathcal{I}, \alpha \models \mathsf{eye}(a) \land \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \forall x (eye(x) \rightarrow
red(x)? Yes
```

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### Questions:

$$\begin{array}{lcl} \Theta &=& \left\{ \begin{array}{ll} \operatorname{eye}(a), \operatorname{eye}(b) \\ \forall x (\operatorname{eye}(x) \to \operatorname{red}(x)) \end{array} \right\} \\ \mathcal{D} &=& \left\{ d_1, \ldots, d_n, \right\} \ n > 1 \\ \operatorname{a}^{\mathcal{I}} &=& d_1 \\ \operatorname{b}^{\mathcal{I}} &=& d_1 \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 \right\} \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} \\ \alpha &=& \left\{ (x \mapsto d_1), (y \mapsto d_2) \right\} \end{array}$$

```
\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \text{eve}(x) \rightarrow
eye(x) \lor eye(y)? Yes
\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow
eye(y)? No
\mathcal{I}, \alpha \models \mathsf{eye}(a) \land \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \forall x (eye(x) \rightarrow
red(x)? Yes
\mathcal{I}, \alpha \models \Theta? Yes
```

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### Questions:

$$egin{array}{lcl} \Theta &=& \left\{ egin{array}{ll} \operatorname{eye}(a),\operatorname{eye}(b) & & & & \\ orall x(\operatorname{eye}(x) 
ightarrow \operatorname{red}(x)) & & & \\ \mathcal{D} &=& \left\{ d_1,\ldots,d_n, 
ight\} & n > 1 & & \\ \operatorname{a}^{\mathcal{I}} &=& d_1 & & \\ \operatorname{b}^{\mathcal{I}} &=& d_1 & & \\ \operatorname{eye}^{\mathcal{I}} &=& \left\{ d_1 
ight\} & & & \\ \operatorname{red}^{\mathcal{I}} &=& \mathcal{D} & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ \end{array}$$

```
\mathcal{I}, \alpha \models \mathsf{eye}(b) \vee \neg \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \text{eve}(x) \rightarrow
eye(x) \lor eye(y)? Yes
\mathcal{I}, \alpha \models \mathsf{eye}(x) \rightarrow
eye(y)? No
\mathcal{I}, \alpha \models \mathsf{eye}(a) \land \mathsf{eye}(b)?
Yes
\mathcal{I}, \alpha \models \forall x (eye(x) \rightarrow
red(x)? Yes
\mathcal{I}, \alpha \models \Theta? Yes
```

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### $\mathcal{I}, \alpha$ is a model of $\varphi$ iff

$$\mathcal{I}, \alpha \models \varphi.$$

A formula can be satisfiable, unsatisfiable, falsifiable, valid. Two formulae  $\varphi$  and  $\psi$  are logically equivalent ( $\varphi \equiv \psi$ ) iff for all  $\mathcal{I}, \alpha$ :

$$\mathcal{I}, \alpha \models \varphi \text{ iff } \mathcal{I}, \alpha \models \psi.$$

Note:  $P(x) \not\equiv P(y)!$ 

Logical Implication is also similar to propositional logic

$$\Theta \models \varphi$$
 iff for all  $\mathcal{I}, \alpha$  s.t.  $\mathcal{I}, \alpha \models \Theta$  also  $\mathcal{I}, \alpha \models \varphi$ .

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Free and bound variables Open and Closed

 $\mathcal{I}, \alpha$  is a model of  $\varphi$  iff

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 iff for all  $\mathcal{I}, \alpha$  s.t.  $\mathcal{I}, \alpha \models \Theta$  also  $\mathcal{I}, \alpha \models \varphi$ 

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Free and bound variables Open and Closed

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$$\Theta \models \varphi$$
 iff for all  $\mathcal{I}, \alpha$  s.t.  $\mathcal{I}, \alpha \models \Theta$  also  $\mathcal{I}, \alpha \models \varphi$ .

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Formulae

 $\mathcal{I}, \alpha$  is a model of  $\varphi$  iff

$$\mathcal{I}, \alpha \models \varphi.$$

A formula can be satisfiable, unsatisfiable, falsifiable, valid. Two formulae  $\varphi$  and  $\psi$  are logically equivalent ( $\varphi \equiv \psi$ ) iff for all  $\mathcal{I}, \alpha$ :

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Logical Implication is also similar to propositional logic

$$\Theta \models \varphi$$
 iff for all  $\mathcal{I}, \alpha$  s.t.  $\mathcal{I}, \alpha \models \Theta$  also  $\mathcal{I}, \alpha \models \varphi$ .

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variables

 $\mathcal{I}, \alpha$  is a model of  $\varphi$  iff

$$\mathcal{I}, \alpha \models \varphi.$$

A formula can be satisfiable, unsatisfiable, falsifiable, valid. Two formulae  $\varphi$  and  $\psi$  are logically equivalent ( $\varphi \equiv \psi$ ) iff for all  $\mathcal{I}, \alpha$ :

$$\mathcal{I},\alpha\models\varphi \ \text{ iff } \ \mathcal{I},\alpha\models\psi.$$

Note:  $P(x) \not\equiv P(y)!$ 

Logical Implication is also similar to propositional logic:

$$\Theta \models \varphi$$
 iff for all  $\mathcal{I}, \alpha$  s.t.  $\mathcal{I}, \alpha \models \Theta$  also  $\mathcal{I}, \alpha \models \varphi$ .

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## Variables can be free or bound (by a quantifier) in a formula:

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\begin{array}{rcl} & \operatorname{free}(x) & = & \{x\} \\ \operatorname{free}(f(t_1,\ldots,t_n)) & = & \operatorname{free}(t_1) \cup \ldots \cup \operatorname{free}(t_n) \\ \operatorname{free}(P(t_1,\ldots,t_n)) & = & \operatorname{free}(t_1) \cup \ldots \cup \operatorname{free}(t_n) \\ & \operatorname{free}(\neg\varphi) & = & \operatorname{free}(\varphi) \\ & \operatorname{free}(\varphi * \psi) & = & \operatorname{free}(\varphi) \cup \operatorname{free}(\psi) * = \vee, \wedge, \rightarrow, \leftrightarrow \\ & \operatorname{free}(\Xi x \varphi) & = & \operatorname{free}(\varphi) - \{x\} \; \Xi = \forall, \exists \end{array}
```

Example:  $\forall x \ (R(y,z) \land \exists \ y \ (\neg P(y,x) \lor R(y,z)))$ Framed occurrences are free, all others are bound Classical Logic

First Order Logic

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Free and bound variables

Variables can be free or bound (by a quantifier) in a formula:

```
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Example:  $\forall x \ (R(y,z) \land \exists \ y \ (\neg P(y,x) \lor R(y,z)))$ Framed occurrences are free, all others are bound Classical Logic

First Order Logic

Semantics

Free and bound variables
Open and Closed

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First Order Logic

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Literature

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First Order Logic

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Free and bound variables

Open and Closed

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First Order Logic

Motivation

Semantics

Free and bound variables
Open and Closed

- Formulae without free variables are called closed formulae or sentences. Formulae with free variables are called open formulae.
- Closed formulae are all we need when we want to state something about the world. Open formulae (and variable maps) are only necessary for technical reasons (semantics of ∀ and ∃).
- Note that logical equivalence, satisfiability, and entailment are independent from variable maps if we consider only closed formulae.
- For closed formulae, we omit  $\alpha$  in connection with  $\models$ :

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Semantics

Free and bound variables

Open and Closed Formulae



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Formulae Literature

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Syntax

Free and bound variables Open and Closed Formulae



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