Exercise 1 (2 points)

1. Is the following set of probabilistic rules a PCFG? Why or why not?

   Terminals: \{you, this, can, see, read, and, should\}
   Nonterminals: \{S, AUX, NP, VP, C, V\}
   Start symbol: S

   Rules and Probabilities:
   S \rightarrow NP VP (0.5)
   S \rightarrow AUX NP VP (0.2)
   S \rightarrow NP AUX VP (0.3)
   VP \rightarrow V NP (1.0)
   NP \rightarrow NP (0.8)
   NP \rightarrow NP C NP (0.2)
   NP \rightarrow you (0.6)
   NP \rightarrow this (0.4)
   AUX \rightarrow can (0.7)
   AUX \rightarrow should (0.3)
   V \rightarrow see (0.9)
   V \rightarrow read (0.1)
   C \rightarrow and (1.0)

2. Find a PCFG in CNF that can generate the same sentences.

In correspondence with forward and backward probabilities in HMMs
   Forward probability \( \alpha_i(t) = P(w_{1(t-1)}; X_t = i) \)
   Backward probability \( \beta_i(t) = P(w_T|X_t = i) \),

   we defined similar concepts for the more general case of PCFGs:
   Outside probability \( \alpha_j(p, q) = P(w_{1(p-1)}; N_{pq}^j; w_{(q+1)m}; G) \)
   Inside probability \( \beta_j(p, q) = P(w_{pq}|N_{pq}^j; G) \).

The inside probability \( \beta_j(p, q) \) is the total probability of generating words \( w_p \ldots w_q \) given that one is starting off with the nonterminal \( N_j \). The outside probability \( \alpha_j(p, q) \) is the total probability of beginning with the start symbol \( N^1 \) and generating the non-terminal \( N_{pq}^j \) and all the words outside \( w_p \ldots w_q \).

In the next two exercises, you use these probabilities.
Exercise 2 (4 points)

Using a parse triangle like in the lecture slide 137, calculate the outside probabilities for the sentence

astronomers saw stars with ears

according to the simple PCFG introduced in the lecture (slide 121). Start at the top righthand corner and work towards the diagonal.

Exercise 3 (6 points)

1. Using the inside and outside probabilities for the sentence

astronomers saw stars with ears

worked out in the lecture and in the previous exercise, reestimate the probabilities of the simple PCFG (slide 121) by working through one iteration of the Inside-Outside algorithm. It is helpful to first link up the inside probabilities shown in slide 137 with the particular rules and subtrees used to obtain them.

2. What would the rule probabilities converge to with continued iterations of the Inside-Outside algorithm? why?