Advanced AI Techniques (WS04)

Exercise sheet 14 (Game Theory)
Deadline: Wednesday (!), 16 Feb 05

Exercise 1 (Kitchen Cleaning Game, 4 points)

Consider the following n person game: Kitchen Cleaning.

Let n people live in a common household (shared appartment?).The kitchen is untidy and dirty. Each person makes its own decision whether s/he cleans the kitchen. The "reward" for the decision to clean the kitchen is -1 (because it takes time and isn't fun). If at least one person decides to clean the kitchen, everybody profits from having a clean kitchen (+1 for everybody). For the person(s) who cleaned the kitchen, the negative and positive rewards are balanced out to a total payoff of 0. If nobody cleans the kitchen, everybody gets a payoff of 0 (we can just tolerate an untidy kitchen).

a. Formulate this problem in terms of game theory. Give a payoff matrix for the cases $n = 2, n = 3$. For arbitrary n, is there a sequence of iterated elimination of weakly dominated strategies leading to a solution, and is there a Nash equilibrium?

b. Now, assume that whenever at least two people decide to clean the kitchen, the "reward" for this decision is $+2$ because working together is more fun and less work. Reformulate the game by adjusting the payoff matrix and determine for the new situation if there is a solution from iterated elimination of strictly dominated strategies, and state if there is a Nash equilibrium.

Exercise 2 (4 points)

Give a proof for the claim:

If there exists a unique solution from iterated elimination of strictly dominated strategies, then it is a Nash equilibrium.

Can it happen that the solution, i.e. action profile, is not unique?

Exercise 3 (Second price auction game, 4 points)

In an auction game, an object is to be assigned to a player in the set $\{1, \ldots n\}$ in exchange for payment. Player i's valuation of the object is v_i, and we assume $v_1 > v_2 > \ldots > 0$. The mechanism used to assign the obect is a sealed-bid auction: The players simultaneously submit bids (nonnegative numbers), and the winner is the person with the highest bid, s/he gets the object. (If several equal highest bids are *made, the person with the lowest index number is the winner.) In our "second price auction", the payment that the winner makes is the highest bid amoung those submitted by the players who do not win (so that if only one player submits the highest bid, then the price paid is the second highest bid).*

Show that in a secound price auction the bid v_i *of any player* i *is a weakly dominant action: player* i*'s payoff when he bids* v_i *is at least as high as his payoff when he submits any other bid, regardless of the actions of the other players. Show that nevertheless there are "inefficient" euqilibria in which the winner is not player 1.*