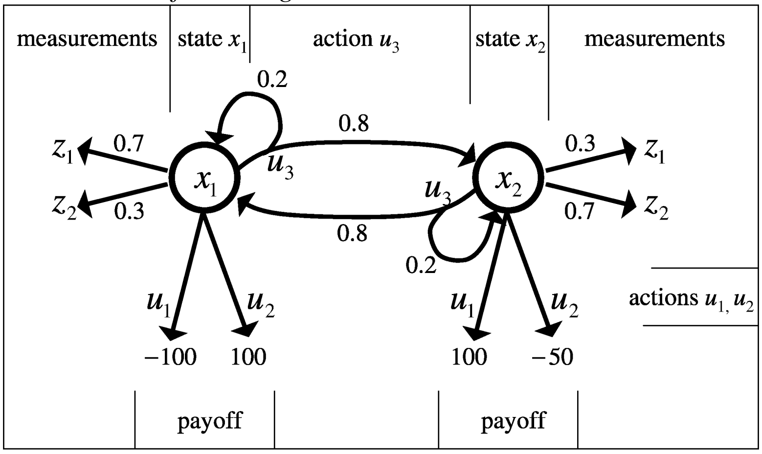
Advanced AI Techniques (WS04)

Exercise sheet 12
Deadline: Thursday, 3 Feb 05

Exercise 1 (3 points)

Consider the following Hidden Markov Model.



Suppose the current belief for being in state x_1 is 0.5 and for being in x_2 is also 0.5. Suppose the agent makes the following sequence of observations: z_1, z_2, z_1. What will be the resulting belief? Now suppose that the agent performs an action u_3 (u_1 and u_2 are terminal actions not considered here) after perceiving z_1 and before perceiving z_1, again. What is the resulting belief in this case?
The diagram reads as follows: $p(x_2 \mid x_1, u_3) = 0.8$ and $p(z_1 \mid x_2) = 0.7$.

Exercise 2 (3 points)

A robot uses a range sensor that can measure ranges from $0m$ up to $3m$. For simplicity, assume that actual ranges are distributed uniformly in this interval. Unfortunately, the sensor can be faulty. When the sensor is faulty, it constantly outputs a range below $1m$, regardless of the actual range in the sensor's measurement cone. We know that the prior probability for a sensor to be faulty is $p = 0.01$.

Suppose the robot queried its sensor N times, and every single time the measurement value is below $1m$. What is the posterior probability of a sensor fault, for $N = 1, 2, \ldots, 10$? Formulate the corresponding probabilistic model.

Exercise 3 (6 points)

A robot has knowledge about its environment as shown in the picture below. It has a prior belief to be in each of the location-direction combinations A, B, C, or D with equal probability of 0.25. The robot has a size of $1m \times 1m$, it exactly fits into a field of the grid. Its program tells it to move $2m$ forward in the next step, but on average, in 1 of 26 cases, its energy is low, then it moves $1m$ instead of $2m$ in such a step.

To track its position, the robot perceives some laser range sensor values z, then it performs a program step (intends a move forward by $2m$), but its odometric information tells it that it moved $1m$, then it receives new range values z'. The range values are indicated next to the picture.

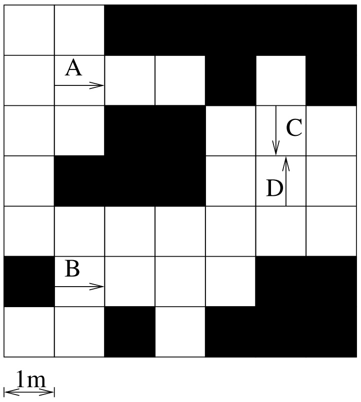
The robot knows that its senses are inaccurate. In $\frac{2}{3}$ of all cases, the odometry is correct, otherwise it counts half of the distance actually moved. All laser range values are given as mulitples of 1m. For each of the four directions, the laser range is correct with probability $p = 0.6$, but with $p = 0.2$, it overestimates the actual distance by $1m$, and with $p = 0.2$, it underestimates the distance (if possible) by $1m$.[1] The measurements for different directions are stochastically independent.

(a.) First, calculate the probabilities $p(z|A), p(z|B)$, etc. Given the range values z, what is the robot's posterior belief of its initial location and walking direction? Why is it not necessary to know the unconditional probability for the range values z?

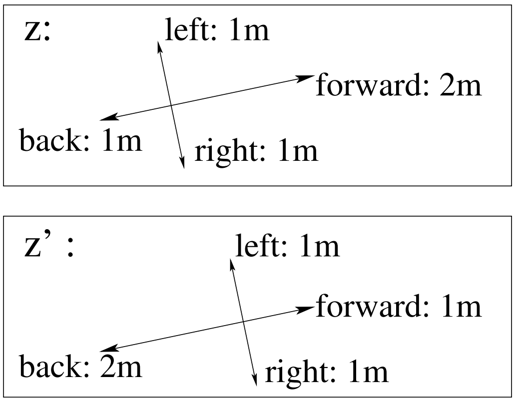
(b.) Derive the probability that the robot actually moved $1m$, given the odometric information, and derive the analogous probability for $2m$.

(c.) Update the robot's belief about its position and direction after the move based on the probabilities of (b.) and the new range sensor data z'.

The environment



The laser range data



[1]For example, in position B, the distance to the left would be measured as $1m$ with probability $p = 0.6$, and $2m$ or $0m$ with $p = 0.2$ each. The distance forward would be measured as $3m$ with $p = 0.6$, and $4m$ or $2m$ with $p = 0.2$ each. The distance backwards would be measured as $0m$ with $p = 0.8$ and $1m$ with $p = 0.2$.