Game Theory

1. Introduction

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What is Game Theory?
Consider rationally acting agents:

Rational agents maximize their (expected) utility:
- decision theory
- Markov decision processes (MDPs)
- reinforcement learning
- AI planning
- ...
Rational Agents in Game Theory

Situation in *game theory*:
Rational Agents in Game Theory

Multiple rational agents interacting in strategic decision situations.

- resulting utility depends on what other agents do.
- all agents know that other agents are rational (this is even common knowledge).

Interesting questions:

- how to model such strategic situations
- how to solve such strategic situations
- how to design games that have desired solutions

Game theory is the study and analysis of such strategic decision situations.
History of Game Theory

- originally part of mathematics and theoretical economics
- today ubiquitous
- here: artificial intelligence and computer science perspective
  - rationality assumptions ("homo economicus") more warranted for artificial agents than for humans
  - interesting algorithmic questions
Application Examples
Two-player board and card games:

- very special
- whatever is good for one player is bad for the other (strictly competitive games)
- recent visible success: Poker (no-limit, heads-up, hold’em)
Auctions

**Auctions**: Think of eBay, Google AdWords, …

- **setting**: one object should be allocated to one out of a number of bidders.
- **questions**:
  - what bidding protocol to use?
  - who is the **winner**?
  - what does the winning bidder have to pay?
Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road

Question: Assume that there are \( n = 2 \) agents. Which routes will they choose?

Average travel cost per agent: ?
Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road

Question: Assume that there are \( n = 2 \) agents. Which routes will they choose?

Average travel cost per agent: 1.5
Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road

Question: Assume that there are \( n = 2 \) agents. Which routes will they choose now (with free new road)?

Average travel cost per agent: ?
Congestion Games

Congestion games: road network with travel costs dependent on the number of agents choosing a particular road.

Question: Assume that there are $n = 2$ agents. Which routes will they choose now (with free new road)?

Average travel cost per agent: $2 > 1.5$
Security Games

Security games:

- **setting**: a facility (e.g., an airport) has to be guarded to avoid attacks
- **possible methods**: visit all critical places, choose the places probabilistically, find a probability distribution for the routing that minimizes expected damage even under the assumption that the attacker can observe the guards
Elections

- **setting**: a set of alternatives (candidates) and a set of voters, determine winner or ranking

- **questions**:
  - what questions to ask?
  - how to determine a winner / ranking?
  - what is the computational complexity of determining a winner?
  - can the protocol be made manipulation-safe?
Rationality
Rationality:

- **General assumption:** All players want to maximize their own utility and nothing else.

- **Contrasts:**
  - Altruistic agents want to maximize utility of other agents
  - Cooperative agents want to maximize group utility
  - Byzantine agents want to minimize utility of other agents

Limitations:

- Agents may not foresee all consequences of their decisions *(bounded rationality)*
- Agents may not know all relevant information about the game structure *(incomplete information)*
- Agents may not know all relevant information about the current state of the game *(imperfect information)*
Course Outline
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- strategic games
- extensive games
- repeated games
- imperfect information games and Poker
- social choice theory
- mechanism design
Let’s Play a Game
We play a game called “Beauty Contest”.

**Rules**

Everybody chooses a natural number $n$ with $1 \leq n \leq 100$. The players that come closest to $\frac{2}{3}$ of the average win.

Now it's your turn!