Game Theory

1. Introduction

Rational Agents

Consider rationally acting agents:

- act
- observe

Rational agents maximize their (expected) utility:
- decision theory
- Markov decision processes (MDPs)
- reinforcement learning
- AI planning
- ...

Rational Agents in Game Theory

Situation in game theory:
Rational Agents in Game Theory

Multiple rational agents interacting in strategic decision situations.

- resulting utility depends on what other agents do.
- all agents know that other agents are rational (this is even common knowledge).

Interesting questions:
- how to model such strategic situations
- how to solve such strategic situations
- how to design games that have desired solutions

Game theory is the study and analysis of such strategic decision situations.

History of Game Theory

- originally part of mathematics and theoretical economics
- today ubiquitous
- here: artificial intelligence and computer science perspective
  - rationality assumptions ("homo economicus") more warranted for artificial agents than for humans
  - interesting algorithmic questions

Board and Card Games

Two-player board and card games:
- very special
- whatever is good for one player is bad for the other (strictly competitive games)
- recent visible success: Poker (no-limit, heads-up, hold'em)
**Auctions**

**Auctions:** Think of eBay, Google AdWords, ...

- **setting:** one object should be allocated to one out of a number of bidders.
- **questions:**
  - what bidding protocol to use?
  - who is the winner?
  - what does the winning bidder have to pay?

**Congestion Games**

**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road

\[ \frac{1}{n} \]

**Question:** Assume that there are \( n = 2 \) agents. Which routes will they choose?

**Average travel cost per agent:** 1.5

**Congestion Games**

**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road

\[ \frac{1}{n} \]

**Question:** Assume that there are \( n = 2 \) agents. Which routes will they choose now (with free new road)?

**Average travel cost per agent:** ?
### Congestion Games

**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road.

- $n_1 / n$ (agents on one route)
- $n_2 / n$ (agents on another route)
- $1$ (expected travel cost per agent)

**Question:** Assume that there are $n = 2$ agents. Which routes will they choose now (with free new road)?

**Average travel cost per agent:** $2 > 1.5$

### Security Games

**Security games:**
- **setting:** a facility (e.g., an airport) has to be guarded to avoid attacks
- **possible methods:**
  - visit all critical places
  - choose the places probabilistically
  - find a probability distribution for the routing that minimizes expected damage even under the assumption that the attacker can observe the guards

### Elections

- **setting:** a set of alternatives (candidates) and a set of voters, determine winner or ranking
- **questions:**
  - what questions to ask?
  - how to determine a winner / ranking?
  - what is the computational complexity of determining a winner?
  - can the protocol be made manipulation-safe?
Rationality

Rationality:
- **General assumption**: All players want to maximize their own utility and nothing else.
- **Contrasts**:
  - **Altruistic** agents want to maximize utility of other agents
  - **Cooperative** agents want to maximize group utility
  - **Byzantine** agents want to minimize utility of other agents

Limitations:
- agents may not foresee all consequences of their decisions (**bounded rationality**)
- agents may not know all relevant information about the game structure (**incomplete information**)
- agents may not know all relevant information about the current state of the game (**imperfect information**)

Course Outline

- strategic games
- extensive games
- repeated games
- imperfect information games and Poker
- social choice theory
- mechanism design

4 Course Outline

- Let's Play a Game

5 Let's Play a Game
We play a game called “Beauty Contest”.

**Rules**

Everybody chooses a natural number $n$ with $1 \leq n \leq 100$. The players that come closest to $2/3$ of the average win.

Now it's your turn!