Exercise 9.1 (Action emulation, 2+2 points)
Consider the following action models (for the single-agent case):

\[ M_1 = e_1 : p \lor q, \]
\[ M_2 = e_2 : p, e_3 : q, \]
\[ M_3 = e_4 : p, e_5 : q, e_6 : p \lor q, \]

(a) Find a total emulation between \( M_1 \) and \( M_3 \). Show that the conditions (forth), (back) and (pre) are satisfied.

(b) Find a total emulation between \( M_2 \) and \( M_3 \). Show that the conditions (forth), (back) and (pre) are satisfied.

Exercise 9.2 (Bisimilarity and action properties, 2+2+2+1+1 points)
Consider the following epistemic state \( s_0 \), alongside the \( Mayset \) actions from Exercise 8.2. The state represents the situation where no agent has pressed the switch yet, but both agents have had the opportunity to do so.

\[ s_0 = w_1 : \neg p \]
\[ w_2 : p \]
\[ w_3 : p \]
\[ w_4 : p \]

We say that an action \( a \) is self-absorbing if for all states \( s \) in which \( a \) is applicable, \( a \) is also applicable in \( s \otimes a \), and \( s \otimes a \otimes a \) is bisimilar to \( s \otimes a \). We say that two actions \( a_1 \) and \( a_2 \) commute if for all states \( s \) where \( a_1 \) is applicable in \( s \) and \( a_2 \) is applicable in \( s \otimes a_1 \), also \( a_2 \) is applicable in \( s \), and \( a_1 \) is applicable in \( s \otimes a_2 \), and \( s \otimes a_1 \otimes a_2 \) is bisimilar to \( s \otimes a_2 \otimes a_1 \).

(a) Show that \( s_0 \), and \( s_0 \otimes Mayset_1' \), and \( s_0 \otimes Mayset_2' \) are bisimilar.

(b) Show that \( Mayset_1 \) and \( Mayset_2 \) are self-absorbing actions.

(c) Show that \( Mayset_1 \) and \( Mayset_2 \) commute.

(d) Give an example for an action that is not self-absorbing.

(e) Give an example for actions that do not commute.