Dynamic Epistemic Logic
3. Public Announcements

So far: Only static knowledge
(Or, where knowledge changed over time, we discussed this change only intuitively, not formally.)

Now: How to model change of knowledge over time?

Note: Knowledge may change in different ways, e.g., via public or private announcements, by sensing, or by ontic (world-changing) actions that affect knowledge along the way.

This chapter: Only public announcements.

Announcement = public and truthful announcement

Example
I announce the fact: “The sun is shining”.
This announcement makes the fact common knowledge.

This holds for all public announcements of true facts about the world.
It does not generally hold for all public announcements of true statements about knowledge.
Public Announcements

Example (Unsuccessful update)

I announce: “p is true, but Bob does not know it” (p ∧ ¬Kb p).

As Bob hears my announcement, he now knows p, and the announced formula p ∧ ¬Kb p becomes false!

Intuition: How should epistemic models look like before and after?

Before: Only those states survive where the announced formula is true.

After:  

![Diagram showing before and after states]

Example (ctd.)

Anne, Bill and Cath have drawn one card from a stack of three cards, 0, 1, 2. Anne has drawn 0, Bill has drawn 1 and Cath the 2.

Anne says: “I do not have card 1”. (¬1a)

Bill states: “I don’t know Anne’s card”. (¬(Kb 0a ∨ Kb 1a ∨ Kb 2a))

Anne says: “I know Bill’s card”. (Kb 0a ∨ Kb 1a ∨ Kb 2a)

Anne says: “I have 0, Bill has 1, Cath has 2.” (0a ∧ 1b ∧ 2c)

Hexa:

![Hexadecimal diagram]

Hexa: 012 ⊨ K¬1a
Hexa: 012 ⊨ K¬(Kb 0a ∨ Kb 1a ∨ Kb 2a)

Announcement Syntax
Announcement Semantics

Recall that, for models $\mathcal{M}$ with domain $S$ and formulas $\varphi$, we write $[\varphi]_\mathcal{M} = \{ s \in S \mid \mathcal{M}, s \models \varphi \}$.

**Definition**

Let $\mathcal{M} = (S, R, V)$ be an epistemic model and $\varphi$ a formula. Then $\mathcal{M}\models \varphi = (S', R', V')$ with

- $S' = [\varphi]_\mathcal{M}$,
- $R'_a = R_a \cap (S' \times S')$ for all $a \in A$, and
- $V'(p) = V(p) \cap S'$.

Example

In (Hexa,012), after Anne announces $\neg 1_a$, Cath knows that $0_a$:

$$\text{Hexa},012 \models [\neg 1_a] K_c 0_a$$

After Bill’s announcement that he does not know Anne’s card, Anne knows Bill’s card:

$$\text{Hexa},012 \models [\neg 1_a][\neg (K_b 0_a \lor K_b 1_a \lor K_b 2_a)] K_a 1_b$$

or: $\text{Hexa}',012 \models [\neg 1_a][\neg (K_b 0_a \lor K_b 1_a \lor K_b 2_a)] K_a 1_b$
Public Announcements
Semantics

Definition
The truth of an $L_K$ (or $L_{KC}$) formula $\varphi$ in an epistemic state $(M, s)$, symbolically $M, s \models \varphi$, is defined as for $L_K$ (or $L_{KC}$), with an additional clause for public announcements:

$M, s \models \square \psi$    iff    ($M, s \models \varphi$ implies $M \models \varphi, s \models \psi$).

Note: $\square \psi$ is satisfied in $s$ if $\varphi$ is not satisfied in $s$.

The dual $\langle \varphi \rangle \psi = \neg \square \neg \psi$ has the truth condition

$M, s \models \varphi$ and $M \models \varphi, s \models \psi$.

Public Announcements
Announcements vs. Revelations

Question: Who actually makes the announcement?
- One of the agents?
- An omniscient external entity?

Observation: This makes a difference!
- If agent $a$ announces $\varphi$, she must know $\varphi$, and could also announce $K_a \varphi$. This can make a difference!
- If the announcement comes from the outside, it is just $[\varphi]$. This is also called a revelation.

Announcements and Revelations

Principles of Public Announcement Logics
Principles of Public Announcement Logics

**Motivation:** In this section, we will prove some valid formulas of the language $\mathcal{L}_{K[]}$ that will ultimately allow us to reduce $\mathcal{L}_{K[]} $ to $\mathcal{L}_{K}$ and get rid of announcement modalities.

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**Proposition (Functionality)**

It is valid that $\langle \varphi \rangle \psi \rightarrow [\varphi] \psi$.

**Proof.**

Let $M, s$ be arbitrary. Assume that $M, s \models \langle \varphi \rangle \psi$.

This is true if and only if $M, s \models \varphi$ and $M, s \models \psi$.

This implies that $M, s \models \varphi$ implies $M, s \models \psi$, i.e., $M, s \models [\varphi] \psi$.

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**Question:** What about the opposite direction? Is $[\varphi] \psi \rightarrow \langle \varphi \rangle \psi$ also valid?

**Proposition**

$[\varphi] \psi \rightarrow \langle \varphi \rangle \psi$ is not valid.

**Proof.**

Counterexample: model $M$ with a single state $s$ where atom $p$ is false. Then $M, s \models [p]p$, but $M, s \not\models \langle p \rangle p$.

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**Proposition (Partiality)**

$\langle \varphi \rangle \top$ is not valid.

**Proof.**

In any epistemic state $M, s$ with $M, s \not\models \varphi$, we have $M, s \not\models \langle \varphi \rangle \top$. 

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Proposition (Negation)

\[ [\varphi] \neg \psi \leftrightarrow (\varphi \rightarrow \neg [\varphi] \psi) \text{ is valid.} \]

Proof.

Omitted. Note that the biimplication can be equivalently written as \([\varphi] \neg \psi \leftrightarrow (\neg \varphi \lor (\varphi) \neg \psi)\).  

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Proposition (Composition)

\[ [\varphi] [\psi] \chi \text{ is equivalent to } [\varphi \land [\varphi] \psi] \chi. \]

Proof.

For arbitrary \(\mathcal{M}, s\), we have \(s \in \mathcal{M}|_{\varphi} [\varphi] \psi\) if \(\mathcal{M}, s \models [\varphi] \psi\) if \(\mathcal{M}, s \models \varphi \land [\varphi] \psi\) if \(\mathcal{M}, s \models \varphi\) and \((\mathcal{M}, s \models [\varphi] \psi)\) if \(s \in \mathcal{M}|_{\varphi}\) and \(\mathcal{M}|_{\varphi}, s \models \psi\) if \(s \in \mathcal{M}|_{\varphi}\).
Principles of Public Announcement Logics

Let us now study how knowledge changes with announcements. We find that $[\varphi]K_a\psi$ is not equivalent to $K_a[\varphi]\psi$.

Counterexample: Hexa, 012 $\vdash [1_a]K_c0_a$, but Hexa, 012 $\not\vdash K_c[1_a]0_a$.

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Proposition (Knowledge)

$[\varphi]K_a\psi$ is equivalent to $\varphi \to K_a[\varphi]\psi$.

Proof.

Assume that $M,s \models \varphi \to K_a[\varphi]\psi$. This holds iff $M,s \models \varphi$ implies $M,s \models K_a[\varphi]\psi$. And $M,s \models \varphi$ implies $M,t \models \psi$ for all $t$ such that $(s,t) \in R_a$.

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Note: Using this proposition, one can reduce any $L_{K[a]}$ formula to an $L_K$ formula. This means that both logics are equally expressive, and that we can use $L_K$ theorem provers or model checkers for $L_{K[a]}$ as well.
Summary

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