Principles of Knowledge Representation and Reasoning

Belief Revision

Bernhard Nebel, Felix Lindner, and Thorsten Engesser

July 5, 2018

Introduction

Belief change

- A dual approach to nonmonotonic reasoning is belief change.
- We start with some belief state K. When new information arrives, we change the belief state in order to accommodate the new information.
- In the general case, the changed belief state may not be a superset of the original belief state.
- Contrary to nonmonotonic reasoning, here we deal with temporal nonmonotonicity, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.

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⇝ Belief revision: change your belief state minimally in order to accommodate the new information

- We have a correct theory about the current state of the world, and the new information is meant to record a change in the world ...

⇝ Belief update: incorporate the change by assuming that the world has changed minimally

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  - Assume we learn that the world has changed and the **door is now closed**.

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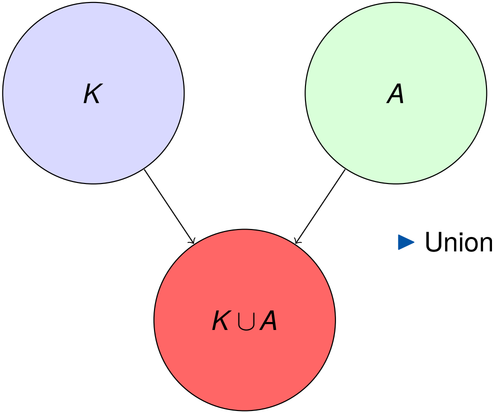
  - Assume we know that the **door is open or the window is open**.
  - Assume we learn that the world has changed and the **door is now closed**.
  - In this case, we do not want to add this information monotonically to our theory, since we would be forced to conclude that **the window is open**.

Belief Revision

Belief revision

- How to react to new information? K is the knowledge base, A some new information



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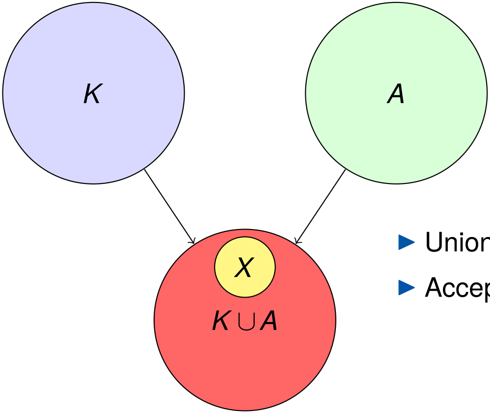

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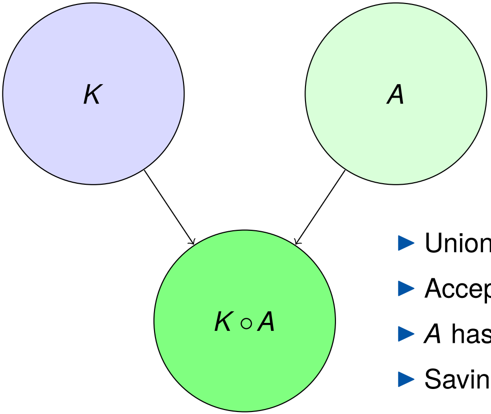

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- Accept loss of beliefs
- A has priority over K
- Saving the most from K

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Belief change operations

General assumption:

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- $\mathrm{Th}_{\mathcal{L}}$: the set of all deductively closed theories (called belief sets) over $\mathcal{L}$

Belief change operations

Most belief change operations have the form:

$$op : \mathrm{Th}_{\mathcal{L}} \times \mathcal{L} \to \mathrm{Th}_{\mathcal{L}}$$

- Expansion: $K + \psi := \mathrm{Cn}(K \cup \{\psi\})$
- Revision: $K \dotplus \varphi$
- Contraction: $K \dot{-} \varphi$ (removal of some belief)

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Harper identity

$$K \dot{-} \varphi \equiv K \cap (K \dotplus \neg\varphi)$$

What is a good revision operator?

Rationale of revision operator:

- Consistency: a revision has to produce a consistent set of beliefs
- Minimality of change: a revision has to change as few beliefs as possible
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To characterize rational revision operators, Alchourron, Gärdenfors, and Makinson identified conditions that should be satisfied by such an operator.

AGM Postulates: Constraining the space of revision operations

AGM postulates:

($\dot{+}$1) $K \dot{+} \varphi \in \mathrm{Th}_{\mathcal{L}}$;

($\dot{+}$2) $\varphi \in K \dot{+} \varphi$;

($\dot{+}$3) $K \dot{+} \varphi \subseteq K + \varphi$;

($\dot{+}$4) If $\neg\varphi \notin K$, then $K + \varphi \subseteq K \dot{+} \varphi$;

($\dot{+}$5) $K \dot{+} \varphi = \mathrm{Cn}(\bot)$ only if $\vdash \neg\varphi$;

($\dot{+}$6) If $\vdash \varphi \leftrightarrow \psi$ then $K \dot{+} \varphi = K \dot{+} \psi$;

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Supplementary postulates:

($\dot{+}$7) $K \dot{+} (\varphi \wedge \psi) \subseteq (K \dot{+} \varphi) + \psi$;

($\dot{+}$8) If $\neg\psi \notin K \dot{+} \varphi$, then $(K \dot{+} \varphi) + \psi \subseteq K \dot{+} (\varphi \wedge \psi)$.

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- The postulates constrain the space to fully rational revision operations, but do not pick a single one.
- Revision operations are closed under intersection, so should we choose the minimum?

Remainder set

Given a belief set K and some new information φ, we are specifically interested in the maximal subtheories consistent with φ:

Definition

Let $A \cup \{\varphi\}$ be a set of formulae. The φ-remainder set of A, denoted by $A \perp \varphi$, is the set of all (inclusion-) maximal subsets B of A that do not entail φ, i.e.:

1. $\varphi \notin Cn(B)$
2. There is no set B' such that $B \subsetneq B' \subseteq A$ with $\varphi \notin Cn(B')$

Canonical revision operations: Full-meet revision

Full-meet contraction/revision

Full-meet contraction: $K \dot{-} \varphi = \bigcap(K \perp \varphi)$ (if $K \perp \varphi \neq \emptyset$; $= K$, else)

Full-meet revision: $K \dot{+} \varphi = (K \dot{-} \neg\varphi) + \varphi$.

- Is full-meet contraction reasonable?

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- Is full-meet contraction reasonable?
- Easy to show: all AGM postulates are satisfied.
- But: it is far too cautious.
  Given φ is inconsistent with K, we get: $K \dot{+} \varphi = \mathrm{Cn}(\varphi)$
- More reasonable: define contraction by only considering some of the remainders: $\rightsquigarrow$ partial meet contraction
- Are there other revision schemes?

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  - the belief set and
  - preference information over this belief set

Examples

Partial meet revision (AGM): Preference information is given by a selection function γ over the set of maximal subtheories consistent with the new information:

$$K \dot{+} \varphi \stackrel{\text{def}}{=} \left(\bigcap \gamma(K \perp \neg\varphi) \right) + \varphi.$$

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Cut revision (GM): Preference information is given by a complete preorder $\preceq$ over all $\psi \in K$:

$$K \dotplus \varphi \stackrel{\text{def}}{=} \{\psi \in K \mid \neg\varphi \prec \psi\} + \varphi.$$

Provided $\preceq$ satisfies a number of axioms (epistemic entrenchment), cut revisions correspond to fully rational revision operations.

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- Consider belief bases (finite sets of propositions) to represent belief sets.
- We don't want to specify an arbitrary amount of preference information …
- A theory K over the propositional logic $\mathcal{L}$ with n propositional atoms can have as much as
  - 2^{2^n} different propositions,
  - 2^n different models.
- Consider ways of specifying preference information in a concise way, i.e., polynomial in the size of the belief base.

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$\Rightarrow$ Base revision schemes generate revision operations in the same way as ordinary schemes do.

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Let $(A \Downarrow \varphi)$ be the maximal subsets of A that are consistent with $\neg\varphi$ and maximize relevant propositions.

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Definition

Let $A \cup \{\varphi\}$ be a set of formulae. The prioritized base-removal $A \Downarrow \varphi$ is the set of all subsets B of A such that:

1. $\varphi \notin Cn(B)$
2. For each $C \subseteq A$ and $1 \leq j \leq n$, if $B \cap \bigcup_{i \geq j} A_i \subsetneq C \cap \bigcup_{i \geq j} A_i$, then $\varphi \in Cn(C \cap \bigcup_{i \geq j} A_i)$.

Note that the 2nd condition is equivalent to:
For each $1 \leq j \leq n$ and each $C \subseteq \bigcup_{i \geq j} A_i$, if $B \cap \bigcup_{i \geq j} A_i \subsetneq C$, then $\varphi \in Cn(C)$.

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Prioritized Meet-Base Revision (PMBR):

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Define a revision operation $\dot{+}$ on $\mathrm{Cn}(A)$ (that depends on A and the priority information) by

$$\mathrm{Cn}(A) \dot{+} \varphi \stackrel{\text{def}}{=} A \oplus \varphi.$$

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$$A = \{p_1, \ldots, p_m, q_1, \ldots, q_m\}, \quad \varphi = \bigwedge_{i=1}^{m} (p_i \leftrightarrow \neg q_i)$$

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  $(A \Downarrow \varphi)$ has size exponential in $|A|$.
- Worse, in some cases there exists no concise representation of the revised base (provided the polynomial hierarchy does not collapse [Cadoli et al 94]).

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- a similar relationship holds between Brewka's level default theories and PMBRs.

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- Rational Monotonicity

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- NMR (at least for default logic) is as hard as belief revision.
- Representing the conclusions from a propositional default theory using classical propositional logic cannot be done in polynomial space, provided the polynomial hierarchy does not collapse.
- In other words, nonmonotonic logics can be thought of representing (some) information in a denser way than classical logic, and with that come higher computational costs.

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Outlook & Summary

- While NMR and Belief Revision seem to be the two sides of the same coin, there are notable pragmatic differences:
  - Belief revision seems to require that we can easily represent the changed belief base, while for NMR it makes sense to use dense representations.
  - A similar argument could be made for the computational complexity.

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- NMR and Belief Revision can be thought of as qualitative ways of dealing with uncertainty in a purely logical setting.
- There exists a strong correspondence between NMR and Belief Revision.
- Both are computationally expensive and representational problematic.
- There are cases, though, that are tractable and practical.

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