

# Principles of Knowledge Representation and Reasoning

## Belief Revision

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# 1 Introduction

- Revision vs. update

# Belief change

- A dual approach to nonmonotonic reasoning is **belief change**.
- We start with some **belief state**  $K$ . When new information arrives, we change the belief state in order to **accommodate the new information**.
- In the general case, the changed belief state may not be a superset of the original belief state.
- Contrary to nonmonotonic reasoning, here we deal with **temporal nonmonotonicity**, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.

# Two scenarios

- We have a theory about the world, and the new information is meant to **correct** our theory ...
  - ↪ **Belief revision**: change your belief state minimally in order to accommodate the new information
- We have a correct theory about the current state of the world, and the new information is meant to record a **change** in the world ...
  - ↪ **Belief update**: incorporate the change by assuming that the world has changed minimally

# Update and revision are different

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Assume the new information is consistent with our old beliefs.

- In case of **belief revision**, we would like to add the new information monotonically to our old beliefs.
- For **belief update** this is not necessarily the case.
  - Assume we know that the **door is open or the window is open**.
  - Assume we learn that the world has changed and the **door is now closed**.
- In this case, we do not want to add this information monotonically to our theory, since we would be forced to conclude that **the window is open**.

# 2 Belief Revision

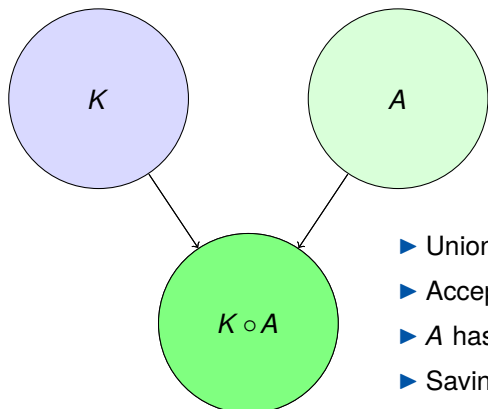
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# Belief revision

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▶ How to react to new information?  $K$  is the knowledge base,  $A$  some new information



- ▶ Union  $\rightarrow$  inconsistency
- ▶ Accept loss of beliefs
- ▶  $A$  has priority over  $K$
- ▶ Saving the most from  $K$

# Belief change operations

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General assumption:

- A **belief state** is modeled by a deductively closed theory, i.e.,  $K = \text{Cn}(K)$  with  $\text{Cn}$  the **consequence operator**
- $\mathcal{L}$ : logical language (propositional logic)
- $\text{Th}_{\mathcal{L}}$ : the set of all deductively closed theories (called **belief sets**) over  $\mathcal{L}$

## Belief change operations

Most belief change operations have the form:

$$op: \text{Th}_{\mathcal{L}} \times \mathcal{L} \rightarrow \text{Th}_{\mathcal{L}}$$

- **Expansion**:  $K + \psi := \text{Cn}(K \cup \{\psi\})$
- **Revision**:  $K \dot{+} \varphi$
- **Contraction**:  $K \dot{-} \varphi$  (removal of some belief)

## Revision vs Contraction

How are revision and contraction related to each other?

Given a contraction operator, one can define a revision operator:

### Levi identity

$$K \dot{+} \varphi \equiv (K \dot{-} \neg\varphi) + \varphi$$

Given a revision operator, one can define a contraction operator:

### Harper identity

$$K \dot{-} \varphi \equiv K \cap (K \dot{+} \neg\varphi)$$

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## What is a good revision operator?

Rationale of revision operator:

- **Consistency**: a revision has to produce a consistent set of beliefs
- **Minimality** of change: a revision has to change as few beliefs as possible
- **Priority** to the new information: the 'new' information is considered more important than the 'old' one

To characterize rational revision operators, Alchourron, Gärdenfors, and Makinson identified conditions that should be satisfied by such an operator.

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## AGM Postulates: Constraining the space of revision operations

### AGM postulates:

- (+1)  $K \dot{+} \varphi \in \text{Th}_{\mathcal{L}}$ ;
- (+2)  $\varphi \in K \dot{+} \varphi$ ;
- (+3)  $K \dot{+} \varphi \subseteq K + \varphi$ ;
- (+4) If  $\neg\varphi \notin K$ , then  $K \dot{+} \varphi \subseteq K + \varphi$ ;
- (+5)  $K \dot{+} \varphi = \text{Cn}(\perp)$  only if  $\vdash \neg\varphi$ ;
- (+6) If  $\vdash \varphi \leftrightarrow \psi$  then  $K \dot{+} \varphi = K \dot{+} \psi$ ;

Supplementary postulates:

- (+7)  $K \dot{+} (\varphi \wedge \psi) \subseteq (K \dot{+} \varphi) + \psi$ ;
- (+8) If  $\neg\psi \notin K \dot{+} \varphi$ , then  $(K \dot{+} \varphi) + \psi \subseteq K \dot{+} (\varphi \wedge \psi)$ .

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## Canonical revision operations?

- AGM postulates do not constrain the operation with respect to varying belief sets!
- The postulates **constrain** the space to **fully rational** revision operations, but do not pick a single one.
- Revision operations are closed under intersection, so should we choose the minimum?

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## Remainder set

Given a belief set  $K$  and some new information  $\varphi$ , we are specifically interested in the **maximal subtheories consistent with  $\varphi$** :

### Definition

Let  $A \cup \{\varphi\}$  be a set of formulae. The  **$\varphi$ -remainder set** of  $A$ , denoted by  $A \perp \varphi$ , is the set of all (inclusion-) maximal subsets  $B$  of  $A$  that do not entail  $\varphi$ , i.e.:

- 1  $\varphi \notin \text{Cn}(B)$
- 2 There is no set  $B'$  such that  $B \subsetneq B' \subseteq A$  with  $\varphi \notin \text{Cn}(B')$

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## Canonical revision operations: Full-meet revision

### Full-meet contraction/revision

**Full-meet contraction:**  $K \dot{-} \varphi = \bigcap (K \perp \varphi)$  (if  $K \perp \varphi \neq \emptyset$ ; =  $K$ , else)

**Full-meet revision:**  $K \dot{+} \varphi = (K \dot{-} \neg \varphi) + \varphi$ .

- Is full-meet contraction reasonable?
- Easy to show: all AGM postulates are satisfied.
- But: it is far too cautious.  
Given  $\varphi$  is inconsistent with  $K$ , we get:  $K \dot{+} \varphi = \text{Cn}(\varphi)$
- More reasonable: define contraction by only considering **some** of the remainders:  $\rightsquigarrow$  **partial meet contraction**
- Are there other revision schemes?

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## Belief revision schemes

- Preference information (what to keep and what to give up)
  - ... may be different for different  $K$ 's, but independent from the new information  $\varphi$
- $\rightsquigarrow$  compose revision operation pointwise for each  $K$
- In general, a **belief revision scheme** (BRS) is a "recipe" for deriving a revision operation – restricted to a particular set  $K$  – from
    - the **belief set** and
    - **preference information** over this belief set

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## Examples

**Partial meet revision** (AGM): Preference information is given by a **selection function**  $\gamma$  over the set of **maximal subtheories consistent** with the new information:

$$K \dot{+} \varphi \stackrel{\text{def}}{=} \left( \bigcap \gamma(K \perp \neg \varphi) \right) + \varphi.$$

**Cut revision** (GM): Preference information is given by a complete preorder  $\preceq$  over all  $\psi \in K$ :

$$K \dot{+} \varphi \stackrel{\text{def}}{=} \{ \psi \in K \mid \neg \varphi \prec \psi \} + \varphi.$$

Provided  $\preceq$  satisfies a number of axioms (**epistemic entrenchment**), cut revisions correspond to **fully rational** revision operations.

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## Revision – Viewed computationally

- We don't want to deal with deductively closed theories ...
- Consider **belief bases** (finite sets of propositions) to **represent** belief sets.
- We don't want to specify an arbitrary amount of preference information ...
- A theory  $K$  over the propositional logic  $\mathcal{L}$  with  $n$  propositional atoms can have as much as
  - $2^{2^n}$  different propositions,
  - $2^n$  different models.
- Consider ways of specifying preference information in a **concise** way, i.e., polynomial in the size of the belief base.

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## Base revision schemes

- Start with a **finite belief base**  $A$  and **preference information** over the elements of  $A$  ...
  - We want to generate a revision operation (restricted to  $Cn(A)$ )
  - Assume a partitioning of  $A$  into  $n$  **priority classes**  $A_1, \dots, A_n$  such that the elements of  $A_i$  are more important or relevant than those of  $A_j$  for  $j < i$
  - Equivalently, consider a complete preorder  $\leq$  over  $A$  comparing priorities (**epistemic relevance**)
  - Define a (**base**) **revision scheme** that keeps as many of the more relevant propositions as possible
- ⇒ Base revision schemes generate revision operations in the same way as ordinary schemes do.

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## Example: Prioritized Meet-Base Revision

Let  $(A \Downarrow \varphi)$  be the maximal subsets of  $A$  that are consistent with  $\neg\varphi$  and **maximize relevant propositions**.

### Definition

Let  $A \cup \{\varphi\}$  be a set of formulae. The **prioritized base-removal**  $A \Downarrow \varphi$  is the set of all subsets  $B$  of  $A$  such that:

- 1  $\varphi \notin Cn(B)$
- 2 For each  $C \subseteq A$  and  $1 \leq j \leq n$ , if  $B \cap \bigcup_{i \geq j} A_i \subsetneq C \cap \bigcup_{i \geq j} A_i$ , then  $\varphi \in Cn(C \cap \bigcup_{i \geq j} A_i)$ .

Note that the 2nd condition is equivalent to:

For each  $1 \leq j \leq n$  and each  $C \subseteq \bigcup_{i \geq j} A_i$ , if  $B \cap \bigcup_{i \geq j} A_i \subsetneq C$ , then  $\varphi \in Cn(C)$ .

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## Example: Prioritized Meet-Base Revision

### Prioritized Meet-Base Revision (PMBR):

$$A \oplus \varphi \stackrel{\text{def}}{=} \left( \bigcap_{B \in (A \Downarrow \neg\varphi)} Cn(B) \right) + \varphi.$$

Define a **revision operation**  $\dagger$  on  $Cn(A)$  (that depends on  $A$  and the priority information) by

$$Cn(A) \dagger \varphi \stackrel{\text{def}}{=} A \oplus \varphi.$$

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## Properties of PMBRs

- Generates **partial meet revision**, but does not satisfy (+8) in general.
- Deciding whether  $A \oplus \varphi \vdash \psi$  is  $\Pi_2^P$ -complete, even for one priority class.
- A **revised base** can be represented by

$$A \oplus \varphi = \text{Cn} \left( (\bigvee (A \Downarrow \neg \varphi)) \wedge \varphi \right).$$

- A revised base can become **exponentially large**:

$$A = \{p_1, \dots, p_m, q_1, \dots, q_m\}, \quad \varphi = \bigwedge_{i=1}^m (p_i \leftrightarrow \neg q_i)$$

$(A \Downarrow \varphi)$  has size exponential in  $|A|$ .

- Worse, in some cases there exists no concise representation of the revised base (provided the polynomial hierarchy does not collapse [Cadoli et al 94]).

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## Revision vs. Nonmonotonic Reasoning

Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there exists a tight connection:

- Given  $K$  and a revision operation  $+$ , a **nonmonotonic consequence relation** can be defined as follows:  $\varphi \sim \psi$  iff  $\psi \in K \dot{+} \varphi$ .

In this case,

- the **rationality postulates** correspond to **principles** of NMR (such as cautious monotonicity, etc.);
- in the case of prerequisite-free, normal defaults  $D$ , the cautious conclusions from  $(W, D)$  are simply  $D \oplus W$  with one priority level;
- a similar relationship holds between **Brewka's level default theories** and **PMBRs**.

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## NMR Principles and Rationality Postulates

- (+2)  $\varphi \in K \dot{+} \varphi$ ;  
  - Reflexivity**
- (+3)  $K \dot{+} \varphi \subseteq K + \varphi$ ;  
  - Supraclassicality**
- (+6) If  $\vdash \varphi \leftrightarrow \psi$  then  $K \dot{+} \varphi = K \dot{+} \psi$ ;  
  - Left Logical Equivalence**
- (+8) If  $\neg \psi \notin K \dot{+} \varphi$ ,  
 then  $(K \dot{+} \varphi) + \psi \subseteq K \dot{+} (\varphi \wedge \psi)$ ;  
  - Rational Monotonicity**

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## Conclusions from the Correspondence

- NMR can be thought of as the other side of the same coin.
- NMR (at least for default logic) is **as hard as** belief revision.
- Representing the conclusions from a propositional default theory using classical propositional logic cannot be done in **polynomial space**, provided the polynomial hierarchy does not collapse.
- In other words, nonmonotonic logics can be thought of representing (some) information in a **denser** way than classical logic, and with that come higher computational costs.

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## Outlook & Summary





- While NMR and Belief Revision seem to be the two sides of the same coin, there are notable **pragmatic differences**:
  - Belief revision seems to require that we can easily represent the changed belief base, while for NMR it makes sense to use **dense representations**.
  - A similar argument could be made for the **computational complexity**.
- NMR and Belief Revision can be thought of as **qualitative ways** of dealing with uncertainty in a purely logical setting.
- There exists a strong **correspondence** between **NMR** and **Belief Revision**.
- Both are computationally expensive and representational problematic.
- There are cases, though, that are tractable and practical.

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## 3 Literature





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