Principles of Knowledge Representation and Reasoning Semantic Networks and Description Logics II: Description Logics – Terminology and Notation

Bernhard Nebel, Felix Lindner, and Thorsten Engesser November 23, 2015 UNI FREIBURG

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- Main problem with semantic networks and frames ... the lack of formal semantics!
- Disadvantage of simple inheritance networks ... concepts are atomic and do not have any structure
- → Brachman's structural inheritance networks (1977)

- Concepts are defined/described using a small set of well-defined operators
- Distinction between conceptual and object-related knowledge
- Computation of subconcept relation and of instance relation
- Strict inheritance (of the entire structure of a concept): inherited properties cannot be overriden

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Systems and applications

Systems:

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- KL-ONE: First implementation of the ideas (1978)
- then: NIKL, KL-TWO, KRYPTON, KANDOR, CLASSIC, BACK, KRIS, YAK, CRACK ...
- later: FaCT, DLP, RACER 1998
- currently: FaCT++, RACER, Pellet, HermiT, and many more

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Applications:

- First, natural language understanding systems,
- then configuration systems,
- and information systems,
- currently, it is one tool for the Semantic Web

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- Languages: DAML+OIL, now OWL (Web Ontology Language)

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Description logics

 Previously also known as KL-ONE-alike languages, frame-based languages, terminological logics, concept languages

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- Previously also known as KL-ONE-alike languages, frame-based languages, terminological logics, concept languages
- Description Logics (DL) allow us
 - to describe concepts using complex descriptions,
 - to introduce the terminology of an application and to structure it (TBox),
 - to introduce objects and relate them to the introduced terminology (ABox),
 - and to reason about the terminology and the objects.

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Informal example

		Motivat
Male is:	the opposite of female	History
		System Applica
A human is a kind of:	living entity	Descrip
A woman is:	a human and a female	a Nutsh
A man is:	a human and a male	Conce Roles
A mother is:	a woman with at least one child that is a human	TBox
A father is:	a man with at least one child that is a human	ABox
A parent is:	a mother or a father	Reaso
A grandmother is:	a woman, with at least one child that is a parent	Servio
A mother-wod is:	a mother with only male children	Outlo
A mother would.		Litera

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Informal example

Male is:	the opposite of female
A human is a kind of:	living entity
A woman is:	a human and a female
A man is:	a human and a male
A mother is:	a woman with at least one child that is a human
A father is:	a man with at least one child that is a human
A parent is:	a mother or a father
A grandmother is:	a woman, with at least one child that is a parent
A mother-wod is:	a mother with only male children

Elizabeth is a woman Elizabeth has the child Charles Charles is a man Diana is a mother-wod Diana has the child William

Possible Questions :

Is a grandmother a parent?
Is Diana a parent?
Is William a man?
Is Elizabeth a mother-wod?

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Concepts and Roles

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Atomic concepts and roles

Concept names:

- E.g., Grandmother, Male, ... (in the following usually capitalized)
- We will use symbols such as *A*,*A*₁,... for concept names
- Semantics: Monadic predicates A(·) or set-theoretically a subset of the universe A^I ⊆ D.

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Role names:

- In our example, e.g., child. Often we will use names such as has-child or something similar (in the following usually lowercase).
- Role names are disjoint from concept names
- Symbolically: *t*,*t*₁,...
- Semantics: Binary relations $t(\cdot, \cdot)$ or set-theoretically $t^{\mathcal{I}} \subseteq \mathcal{D} \times \mathcal{D}$.

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Concept and role description

- From (atomic) concept and role names, complex concept and role descriptions can be created
- In our example, e.g., "Human and Female."
- Symbolically: C for concept descriptions and r for role descriptions

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Concept and role description

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Which particular constructs are available depends on the chosen description logic!

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Concept and role description

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- Symbolically: C for concept descriptions and r for role descriptions

Which particular constructs are available depends on the chosen description logic!

- FOL semantics: A concept description *C* corresponds to a formula *C*(*x*) with the free variable *x*. Similarly with role descriptions *r*: they correspond to formulae *r*(*x*,*y*) with free variables *x*,*y*.
- Set semantics:

$$C^{\mathcal{I}} = \{ d \in \mathcal{D} : C(d) \text{ "is true in" } \mathcal{I} \}$$

 $r^{\mathcal{I}} = \{ (d, e) \in \mathcal{D}^2 : r(d, e) \text{ "is true in" } \mathcal{I} \}$

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Syntax: let *C* and *D* be concept descriptions, then the following are also concept descriptions:

- $C \sqcap D$ (concept conjunction)
- C □ D (concept disjunction)
- $\neg C$ (concept negation)

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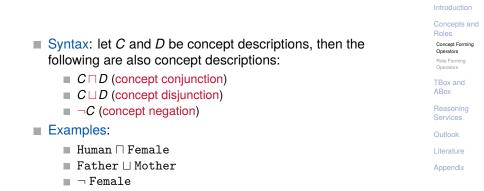
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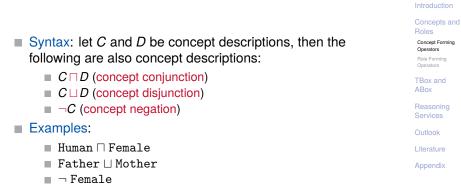
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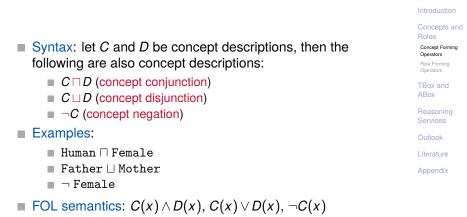






FOL semantics: $C(x) \wedge D(x)$, $C(x) \vee D(x)$, $\neg C(x)$





Set semantics: $C^{\mathcal{I}} \cap D^{\mathcal{I}}, C^{\mathcal{I}} \cup D^{\mathcal{I}}, \mathcal{D} \setminus C^{\mathcal{I}}$

Role restrictions

Motivation: Often we want to describe something by restricting the possible "fillers" of a role, e.g. Mother-wod. Sometimes we want to say that there is at least a filler of a particular type, e.g. Grandmother



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Role restrictions

Motivation:	Introduction
 Often we want to describe something by restricting the 	Concepts an Roles
possible "fillers" of a role, e.g. Mother-wod.	Concept Forming Operators
Sometimes we want to say that there is at least a filler of a	Role Forming Operators
particular type, e.g. Grandmother	TBox and
Idea: Use quantifiers that range over the role-fillers	ABox
■ Mother □ ∀has-child.Man	Reasoning Services
■ Woman □ ∃has-child.Parent	Outlook
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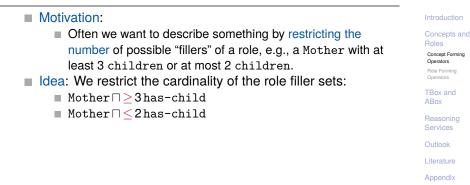
Role restrictions

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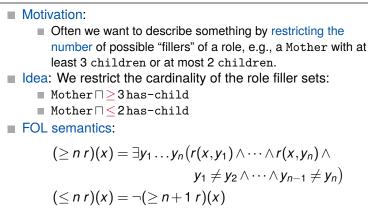
Often we	want to describe sor	mething by restricting the	е	Concepts an Roles
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■ Mother □ ∀has-child.Man			Reasoning Services	
■ Woman □ ∃has-child.Parent			Outlook	
FOL semanti	cs:			Literature
	$(\exists r.C)(x) = \exists y(r(x))$	$(x,y)\wedge C(y))$		Appendix
	$(\forall r.C)(x) = \forall y(r($	$(x,y) \to C(y))$		
Set semantic	s:			
$(\exists r.C)^{\mathcal{I}} = \{c$	$m{\ell}\in\mathcal{D}$: there ex. set	ome <i>e</i> s.t. $(d, e) \in r^{\mathcal{I}}$ with $(d, e) \in r^{\mathcal{I}}$ $e \in G$	$\wedge e \in C^{\mathcal{I}}$	} g
$(orall r. \mathcal{C})^\mathcal{I} = ig c$	$m{\ell}\in\mathcal{D}$: for each $m{e}$	with $({m d},{m e})\in {m r}^{\mathcal I},\ {m e}\in {m C}$	\mathcal{I}	
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Cardinality restriction





Cardinality restriction



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Set semantics:

$$(\geq n r)^{\mathcal{I}} = \left\{ d \in \mathcal{D} : \left| \left\{ e \in \mathcal{D} : r^{\mathcal{I}}(d, e) \right\} \right| \geq n \right\} \\ (\leq n r)^{\mathcal{I}} = \mathcal{D} \setminus (\geq n+1 r)^{\mathcal{I}}$$

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Inverse roles

Motivation:

- How can we describe the concept "children of rich parents"?
- Idea: Define the "inverse" role for a given role (the converse relation)
 - has-child⁻¹
- Example: ∃has-child⁻¹.Rich

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Inverse roles

Motivation:

- How can we describe the concept "children of rich parents"?
- Idea: Define the "inverse" role for a given role (the converse relation)
 - has-child⁻¹
- Example: ∃has-child⁻¹.Rich
- FOL semantics:

$$r^{-1}(x,y)=r(y,x)$$

Set semantics:

$$(r^{-1})^{\mathcal{I}} = \left\{ (d, e) \in \mathcal{D}^2 : (e, d) \in r^{\mathcal{I}}
ight\}$$

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Role composition

Motivation:

- How can we define the role has-grandchild given the role has-child?
- Idea: Compose roles (as one can compose binary relations)
 - has-child o has-child

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Role composition

Motivation:

- How can we define the role has-grandchild given the role has-child?
- Idea: Compose roles (as one can compose binary relations)
 - has-child o has-child
- FOL semantics:

$$(r \circ s)(x,y) = \exists z(r(x,z) \land s(z,y))$$

Set semantics:

$$(r \circ s)^{\mathcal{I}} = \left\{ (d, e) \in \mathcal{D}^2 : \exists f \text{ s.t. } (d, f) \in r^{\mathcal{I}} \land (f, e) \in s^{\mathcal{I}} \right\}$$

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Role value maps

Motivation: How do we express the concept "women who know all the friends of their children" Idea: Relate role filler sets to each other Woman \sqcap (has-child \circ has-friend \sqsubseteq knows) FOL semantics: $(r \sqsubseteq s)(x) = \forall y (r(x,y) \rightarrow s(x,y))$

Set semantics: Let $r^{\mathcal{I}}(d) = \{e : r^{\mathcal{I}}(d, e)\}.$

$$(r \sqsubseteq s)^{\mathcal{I}} = \left\{ d \in \mathcal{D} \, : r^{\mathcal{I}}(d) \subseteq s^{\mathcal{I}}(d) \right\}$$

Note: Role value maps lead to undecidability of satisfiability testing of concept descriptions!

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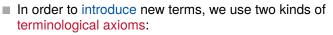
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TBox and ABox

Terminology box



$$A \doteq C$$

where *A* is a concept name and *C* is a concept description.

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Terminology box

In order to introduce new terms, we use two kinds of terminological axioms:

$$A \doteq C$$

$$A \sqsubseteq C$$

where A is a concept name and C is a concept description.

- A terminology or TBox is a finite set of such axioms with the following additional restrictions:
 - no multiple definitions of the same symbol such as $A \doteq C$, $A \sqsubseteq D$
 - no cyclic definitions (even not indirectly), such as $A \doteq \forall r . B$, $B \doteq \exists s . A$

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TBoxes: semantics

TBoxes restrict the set of possible interpretations.

FOL semantics:

- $A \doteq C$ corresponds to $\forall x (A(x) \leftrightarrow C(x))$ $A \sqsubseteq C$ corresponds to $\forall x (A(x) \rightarrow C(x))$

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TBoxes: semantics

TBoxes restrict the set of possible interpretations.

■ FOL semantics:

- $A \doteq C$ corresponds to $\forall x (A(x) \leftrightarrow C(x))$
- $A \sqsubseteq C$ corresponds to $\forall x (A(x) \rightarrow C(x))$

Set semantics:

- $A \doteq C$ corresponds to $A^{\mathcal{I}} = C^{\mathcal{I}}$
- $A \sqsubseteq C$ corresponds to $A^{\mathcal{I}} \subseteq C^{\mathcal{I}}$

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TBoxes: semantics

TBoxes restrict the set of possible interpretations.

FOL semantics:

- $A \doteq C$ corresponds to $\forall x (A(x) \leftrightarrow C(x))$
- $A \sqsubseteq C$ corresponds to $\forall x (A(x) \rightarrow C(x))$

Set semantics:

- $A \doteq C$ corresponds to $A^{\mathcal{I}} = C^{\mathcal{I}}$
- $A \sqsubseteq C$ corresponds to $A^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
- Non-empty interpretations which satisfy all terminological axioms are called models of the TBox.

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Assertional box

In order to state something about objects in the world, we use two forms of assertions:

where a and b are individual names (e.g., ELIZABETH, PHILIP), C is a concept description, and r is a role description.

An ABox is a finite set of assertions.

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ABoxes: semantics

- Individual names are interpreted as elements of the universe under the unique-name-assumption, i.e., different names refer to different objects.
- Assertions express that an object is an instance of a concept or that two objects are related by a role.

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FOL semantics:

- a: C corresponds to C(a)
- (a,b) : r corresponds to r(a,b)
- Set semantics:
 - $\blacksquare a^{\mathcal{I}} \in D$
 - a: C corresponds to $a^{\mathcal{I}} \in C^{\mathcal{I}}$
 - (*a*,*b*) : *r* corresponds to $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$

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Models of an ABox and of ABox + TBox can be defined analogously to models of a TBox.

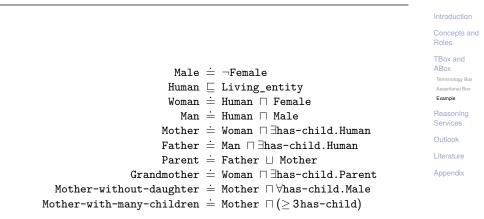
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Example TBox





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CHARLES: Man EDWARD: Man ANDREW: Man		DIANA: Woman ELIZABETH: Woman	TBox and ABox Terminology Box Assertional Box Example
	out-daughter		Reasoning Services
(ELIZABETH, CHARLES):	has-child		Outlook
(ELIZABETH, EDWARD):	has-child		Literature
(ELIZABETH, ANDREW):	has-child		
(DIANA, WILLIAM):	has-child		Appendix
(CHARLES, WILLIAM):	has-child		



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■ Does a description *C* make sense at all, i.e., is it satisfiable? A concept description *C* is satisfiable, if there exists an interpretation *I* such that C^I ≠ Ø. Introduction

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- Does a description *C* make sense at all, i.e., is it satisfiable? A concept description *C* is satisfiable, if there exists an interpretation *I* such that C^I ≠ Ø.
- Is one concept a specialization of another one, is it subsumed?
 C is subsumed by *D* (in symbols *C* ⊆ *D*) if we have for all interpretations C^I ⊂ D^I.

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- Is *a* an instance of a concept *C*? *a* is an instance of *C* if for all interpretations, we have $a^{\mathcal{I}} \in C^{\mathcal{I}}$.
- Note: These questions can be posed with or without a TBox that restricts the possible interpretations.

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- Can we reduce the reasoning services to perhaps just one problem?
- What could be reasoning algorithms?
- What can we say about complexity and decidability?
- What has all that to do with modal logics?
- How can one build efficient systems?

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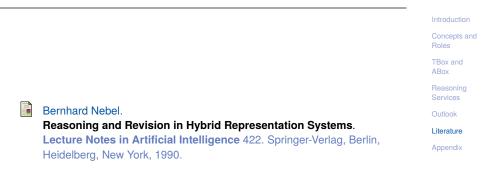
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Literature II





Summary: Concept descriptions

Abstract	Concrete	Interpretation		Introduction
A	Α	$A^{\mathcal{I}}$		Concepts and
$C \sqcap D$	(and <i>C D</i>)	$\mathcal{C}^{\mathcal{I}} \cap \mathcal{D}^{\mathcal{I}}$		Roles
$C \sqcup D$	(or <i>C D</i>)	$\mathcal{C}^{\mathcal{I}} \cup \mathcal{D}^{\mathcal{I}}$		TBox and ABox
$\neg C$	(not <i>C</i>)	$\mathcal{D} - \mathcal{C}^{\mathcal{I}}$		
$\forall r.C$	(all <i>r C</i>)	$\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \subseteq oldsymbol{C}^\mathcal{I} ight\}$		Reasoning Services
$\exists r$	(some r)	$\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}^{\mathcal{I}}(\boldsymbol{d}) \neq \boldsymbol{\emptyset} ight\}^{-1}$		Outlook
\geq n r	(atleast n r)	$\left\{ d \in \mathcal{D} : r^{\mathcal{I}}(d) \geq n \right\}$		Literature
\leq n r	(atmost <i>n r</i>)	$\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}_{-}^{\mathcal{I}}(\boldsymbol{d}) \leq \underline{n} \right\}$		Appendix
∃r.C	(some r C)	$\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}^{\mathcal{I}}(\boldsymbol{d}) \cap \boldsymbol{C}^{\mathcal{I}} \neq \boldsymbol{\emptyset} ight\}$		
\geq n r.C	(atleast n r C)	$\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \cap oldsymbol{C}^\mathcal{I} \geq n ight\}$		
\leq n r.C	(atmost <i>n r C</i>)	$\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \cap oldsymbol{C}^\mathcal{I} \leq n ight\}$		
$r \stackrel{\cdot}{=} s$	(eq <i>r s</i>)	$\left\{ d \in \mathcal{D} : r^{\mathcal{I}}(d) = s^{\mathcal{I}}(d) \right\}$		
$r \neq s$	(neq <i>r s</i>)	$\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}^{\mathcal{I}}(\boldsymbol{d}) \neq \boldsymbol{s}^{\mathcal{I}}(\boldsymbol{d}) ight\}$		
$r \sqsubseteq s$	(subset r s)	$\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \subseteq oldsymbol{s}^\mathcal{I}(oldsymbol{d}) ight\}$		
$g\stackrel{\cdot}{=}h$	(eq <i>g h</i>)	$ig\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{g}^\mathcal{I}(oldsymbol{d}) = oldsymbol{h}^\mathcal{I}(oldsymbol{d}) eq \emptyset ig\}$		13
g eq h	(neq <i>g h</i>)	$\left\{ d \in \mathcal{D} : \emptyset \neq g^{\mathcal{I}}(d) \neq h^{\mathcal{I}}(d) \neq \emptyset \right\}$		NC NC
$\{i_1, i_2, \ldots, i_n\}$	(oneof $i_1 \dots i_n$)	$\{i_1^{\mathcal{I}}, i_2^{\mathcal{I}}, \dots, i_n^{\mathcal{I}}\}$	r	
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Summary: Role descriptions

			Concepts Roles
			TBox and ABox
Abstract	Concrete	Interpretation	Reasonin
t	t	$t^{\mathcal{I}}$	Services
f	f	$f^{\mathcal{I}},$ (functional role)	Outlook
<i>r</i> ⊓ <i>s</i>	(and <i>r s</i>)	$r^{\mathcal{I}} \cap s^{\mathcal{I}}$	Literature
r⊔s	(or <i>r s</i>)	$r^{\mathcal{I}} \cup s^{\mathcal{I}}$	Appendix
$\neg r$	(not <i>r</i>)	$\mathcal{D} imes \mathcal{D} - r^{\mathcal{I}}$	пропак
r ⁻¹	(inverse r)	$egin{array}{llllllllllllllllllllllllllllllllllll$	
$r _C$	(restr r C)		
r ⁺	(trans r)	$(r^{\mathcal{I}})^+$	
<i>r</i> ∘ <i>s</i>	(compose r s)	$r^{\mathcal{I}} \circ s^{\mathcal{I}}$	
1	self	$\{(d,d): d \in \mathcal{D}\}$	



and