Principles of Knowledge Representation and Reasoning Semantic Networks and Description Logics II: Description Logics – Terminology and Notation

Bernhard Nebel, Felix Lindner, and Thorsten Engesser November 23, 2015 UNI FREIBURG

1 Introduction

| | History Systems and Applications Description Logics in |
|--|---|
| MotivationHistory | a Nutshell Concepts and Roles |
| Systems and Applications | TBox and ABox |
| Description Logics in a Nutshell | Reasoning Services |
| | Outlook |
| | Literature |
| | Appendix |



Introduction

Motivation

Introduction

Motivation

History Systems and Applications Description Logics in a Nutshell

Concepts and Roles

TBox and ABox

Reasoning Services

Outlook

Literature





- Main problem with semantic networks and frames ... the lack of formal semantics!
- Disadvantage of simple inheritance networks ... concepts are atomic and do not have any structure
- → Brachman's structural inheritance networks (1977)

- Concepts are defined/described using a small set of well-defined operators
- Distinction between conceptual and object-related knowledge
- Computation of subconcept relation and of instance relation
- Strict inheritance (of the entire structure of a concept): inherited properties cannot be overriden

Introduction

Motivation

History

Systems and Applications Description Logics in a Nutshell

Concepts and Roles

TBox and ABox

Reasoning Services

Outlook

Literature



Systems and applications

Systems:

- KL-ONE: First implementation of the ideas (1978)
- then: NIKL, KL-TWO, KRYPTON, KANDOR, CLASSIC, BACK, KRIS, YAK, CRACK ...
- later: FaCT, DLP, RACER 1998
- currently: FaCT++, RACER, Pellet, HermiT, and many more

Applications:

- First, natural language understanding systems,
- then configuration systems,
- and information systems,
- currently, it is one tool for the Semantic Web
- Languages: DAML+OIL, now OWL (Web Ontology Language)

Introduction

Motivation

listory

Systems and Applications

Description Logics ir a Nutshell

Concepts and Roles

TBox and ABox

Reasoning Services

Outlook

Literature

- Previously also known as KL-ONE-alike languages, frame-based languages, terminological logics, concept languages
- Description Logics (DL) allow us
 - to describe concepts using complex descriptions,
 - to introduce the terminology of an application and to structure it (TBox),
 - to introduce objects and relate them to the introduced terminology (ABox),
 - and to reason about the terminology and the objects.

Introduction

Motivation

History Systems and

Description Logics in

Concepts and Roles

TBox and ABox

Reasoning Services

Outlook

Literature

Informal example

| Male is: | the opposite of female |
|-----------------------|---|
| A human is a kind of: | living entity |
| A woman is: | a human and a female |
| A man is: | a human and a male |
| A mother is: | a woman with at least one child that is a human |
| A father is: | a man with at least one child that is a human |
| A parent is: | a mother or a father |
| A grandmother is: | a woman, with at least one child that is a parent |
| A mother-wod is: | a mother with only male children |

Elizabeth is a woman Elizabeth has the child Charles Charles is a man Diana is a mother-wod Diana has the child William

Possible Questions :

Is a grandmother a parent?
Is Diana a parent?
Is William a man?
Is Elizabeth a mother-wod?

Introduction

Motivation

History

Systems and Applications

Description Logics in a Nutshell

Concepts and Roles

TBox and ABox

Reasoning Services

Outlook

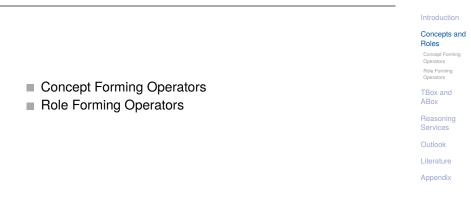
Literature

Appendix

BURG

. .

2 Concepts and Roles





Atomic concepts and roles

Concept names:

- E.g., Grandmother, Male, ... (in the following usually capitalized)
- We will use symbols such as *A*,*A*₁,... for concept names
- Semantics: Monadic predicates A(·) or set-theoretically a subset of the universe A^I ⊆ D.

Role names:

- In our example, e.g., child. Often we will use names such as has-child or something similar (in the following usually lowercase).
- Role names are disjoint from concept names
- Symbolically: *t*,*t*₁,...
- Semantics: Binary relations $t(\cdot, \cdot)$ or set-theoretically $t^{\mathcal{I}} \subseteq \mathcal{D} \times \mathcal{D}$.

Introduction

Concepts and Roles

Concept Forming Operators

Role Forming Operators

TBox and ABox

Reasoning Services

Outlook

Literature

Appendix

BURG

Concept and role description

- From (atomic) concept and role names, complex concept and role descriptions can be created
- In our example, e.g., "Human and Female."
- Symbolically: C for concept descriptions and r for role descriptions

Which particular constructs are available depends on the chosen description logic!

- FOL semantics: A concept description *C* corresponds to a formula *C*(*x*) with the free variable *x*. Similarly with role descriptions *r*: they correspond to formulae *r*(*x*,*y*) with free variables *x*,*y*.
- Set semantics:

$$C^{\mathcal{I}} = \{ d \in \mathcal{D} : C(d) \text{ "is true in" } \mathcal{I} \}$$

 $r^{\mathcal{I}} = \{ (d, e) \in \mathcal{D}^2 : r(d, e) \text{ "is true in" } \mathcal{I} \}$

Introduction

Concepts and Roles

Concept Forming Operators

Role Forming Operators

TBox and ABox

Reasoning Services Outlook Literature

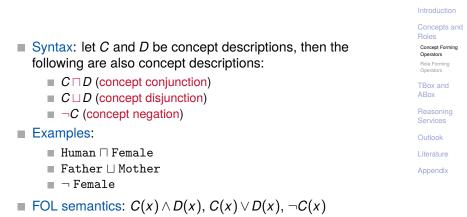
Appendix

12/36

November 23, 2015

Nebel, Lindner, Engesser – KR&R

Boolean operators



Set semantics: $C^{\mathcal{I}} \cap D^{\mathcal{I}}, C^{\mathcal{I}} \cup D^{\mathcal{I}}, \mathcal{D} \setminus C^{\mathcal{I}}$

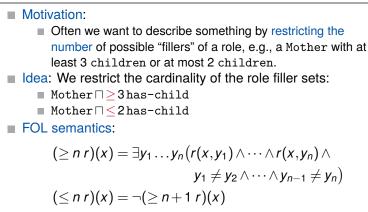
Role restrictions

Mativation

-

| Often we want to describe something by restricting the possible "fillers" of a role, e.g. Mother-wod. Sometimes we want to say that there is at least a filler of a particular type, e.g. Grandmother Idea: Use quantifiers that range over the role-fillers | |
|---|----------------------|
| Sometimes we want to say that there is at least a filler of a particular type, e.g. Grandmother TBox | cepts an s |
| Sometimes we want to say that there is at least a liner of a particular type, e.g. Grandmother Idea: Use quantifiers that range over the role-fillers | ept Forming itors |
| ■ Idea: Use quantifiers that range over the role-fillers | orming itors |
| Idea. Use quantiners that range over the role-inters | and |
| - Mathematical Market Bessel Besse | C |
| ■ Mother □ ∀has-child.Man Reas | soning ices |
| ■ Woman □ ∃has-child.Parent | |
| FOL semantics: | |
| $(\exists r.C)(x) = \exists y(r(x,y) \land C(y))$ | |
| $(\forall r.C)(x) = \forall y (r(x,y) \rightarrow C(y))$ | |
| Set semantics: | |
| Set semantics: $(\exists r.C)^{\mathcal{I}} = \{ d \in \mathcal{D} : \text{ there ex. some } e \text{ s.t. } (d, e) \in r^{\mathcal{I}} \land e \in C^{\mathcal{I}} \}$ $(\forall r.C)^{\mathcal{I}} = \{ d \in \mathcal{D} : \text{ for each } e \text{ with } (d, e) \in r^{\mathcal{I}}, e \in C^{\mathcal{I}} \}$ | |
| $(\forall r.C)^{\mathcal{I}} = \left\{ d \in \mathcal{D} : \text{ for each } e \text{ with } (d, e) \in r^{\mathcal{I}}, \ e \in C^{\mathcal{I}} ight\}$ | |
| November 23, 2015 Nebel, Lindner, Engesser – KR&R 14 / 36 | |

Cardinality restriction



Introduction

Concepts and Roles

Concept Forming Operators

Role Forming Operators

TBox and ABox

Reasoning Services

Outlook

Literature

Appendix

BURG

Set semantics:

$$(\geq n r)^{\mathcal{I}} = \left\{ d \in \mathcal{D} : \left| \left\{ e \in \mathcal{D} : r^{\mathcal{I}}(d, e) \right\} \right| \geq n \right\} \\ (\leq n r)^{\mathcal{I}} = \mathcal{D} \setminus (\geq n+1 r)^{\mathcal{I}}$$

November 23, 2015

Nebel, Lindner, Engesser - KR&R

Inverse roles

Motivation:

- How can we describe the concept "children of rich parents"?
- Idea: Define the "inverse" role for a given role (the converse relation)
 - has-child⁻¹
- Example: ∃has-child⁻¹.Rich
- FOL semantics:

$$r^{-1}(x,y)=r(y,x)$$

Set semantics:

$$(r^{-1})^{\mathcal{I}} = \left\{ (d, e) \in \mathcal{D}^2 : (e, d) \in r^{\mathcal{I}}
ight\}$$

Introduction

Concepts and Roles

Concept Forming Operators

Role Forming Operators

TBox and ABox

Reasoning Services

Outlook

Literature

Appendix

BURG

Role composition

Motivation:

- How can we define the role has-grandchild given the role has-child?
- Idea: Compose roles (as one can compose binary relations)
 - has-child o has-child
- FOL semantics:

$$(r \circ s)(x,y) = \exists z(r(x,z) \land s(z,y))$$

Set semantics:

$$(r \circ s)^{\mathcal{I}} = \left\{ (d, e) \in \mathcal{D}^2 : \exists f \text{ s.t. } (d, f) \in r^{\mathcal{I}} \land (f, e) \in s^{\mathcal{I}} \right\}$$

Introduction

Concepts and Roles

Concept Forming Operators

Role Forming Operators

TBox and ABox

Reasoning Services

Outlook

Literature

Role value maps

Motivation: How do we express the concept "women who know all the friends of their children" Idea: Relate role filler sets to each other Woman \sqcap (has-child \circ has-friend \sqsubseteq knows) FOL semantics: $(r \sqsubseteq s)(x) = \forall y (r(x,y) \rightarrow s(x,y))$

Set semantics: Let $r^{\mathcal{I}}(d) = \{e : r^{\mathcal{I}}(d, e)\}.$

$$(r \sqsubseteq s)^{\mathcal{I}} = \left\{ d \in \mathcal{D} \, : r^{\mathcal{I}}(d) \subseteq s^{\mathcal{I}}(d) \right\}$$

Note: Role value maps lead to undecidability of satisfiability testing of concept descriptions!

November 23, 2015

Nebel, Lindner, Engesser - KR&R

18/36

Appendix

BURG

3 TBox and ABox

| | Introduction |
|---|---|
| | Concepts and Roles |
| Terminology BoxAssertional Box | TBox and ABox Terminology Box Assertional Box Example |
| Example | Reasoning Services |
| | Outlook |
| | Literature |



Terminology box

In order to introduce new terms, we use two kinds of terminological axioms:

$$A \doteq C$$

$$A \sqsubseteq C$$

where A is a concept name and C is a concept description.

- A terminology or TBox is a finite set of such axioms with the following additional restrictions:
 - no multiple definitions of the same symbol such as $A \doteq C$, $A \sqsubseteq D$
 - no cyclic definitions (even not indirectly), such as $A \doteq \forall r . B$, $B \doteq \exists s . A$

Introduction

Concepts and Roles

TBox and ABox

Terminology Box Assertional Box

Reasoning Services

Outlook

Literature

Appendix

TBoxes: semantics

TBoxes restrict the set of possible interpretations.

FOL semantics:

- $A \doteq C$ corresponds to $\forall x (A(x) \leftrightarrow C(x))$
- $A \sqsubseteq C$ corresponds to $\forall x (A(x) \rightarrow C(x))$

Set semantics:

- $A \doteq C$ corresponds to $A^{\mathcal{I}} = C^{\mathcal{I}}$
- $A \sqsubseteq C$ corresponds to $A^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
- Non-empty interpretations which satisfy all terminological axioms are called models of the TBox.

Introduction

Concepts and Roles

TBox and ABox

Terminology Box

Assertional Bo

Reasoning Services

Outlook

Literature

Assertional box

In order to state something about objects in the world, we use two forms of assertions:

where a and b are individual names (e.g., ELIZABETH, PHILIP), C is a concept description, and r is a role description.

An ABox is a finite set of assertions.

Concepts and Roles

TBox and ABox

Assertional Box

Example

Reasoning Services

Outlook

Literature



ABoxes: semantics

- Individual names are interpreted as elements of the universe under the unique-name-assumption, i.e., different names refer to different objects.
- Assertions express that an object is an instance of a concept or that two objects are related by a role.

FOL semantics:

- a: C corresponds to C(a)
- (a,b) : r corresponds to r(a,b)
- Set semantics:
 - $\blacksquare a^{\mathcal{I}} \in D$
 - a: C corresponds to $a^{\mathcal{I}} \in C^{\mathcal{I}}$
 - (*a*,*b*) : *r* corresponds to $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$

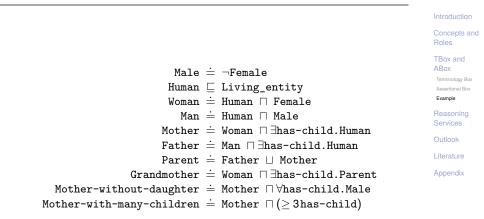
Models of an ABox and of ABox + TBox can be defined analogously to models of a TBox.

Concepts at Roles TBox and ABox Terminology Box Assertional Box Example Reasoning Services Outlook

Literature

Appendix

Example TBox

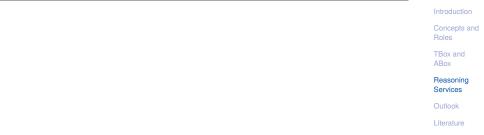




| | | | Introduction |
|--|--------------|----------------------------------|---|
| | | | Concepts and Roles |
| CHARLES: Man EDWARD: Man ANDREW: Man | | DIANA: Woman ELIZABETH: Woman | TBox and ABox Terminology Box Assertional Box Example |
| | out-daughter | | Reasoning Services |
| (ELIZABETH, CHARLES): | has-child | | Outlook |
| (ELIZABETH, EDWARD): | has-child | | Literature |
| (ELIZABETH, ANDREW): | has-child | | |
| (DIANA, WILLIAM): | has-child | | Appendix |
| (CHARLES, WILLIAM): | has-child | | |



4 Reasoning Services





Some reasoning services

- Does a description *C* make sense at all, i.e., is it satisfiable? A concept description *C* is satisfiable, if there exists an interpretation *I* such that C^I ≠ Ø.
- Is one concept a specialization of another one, is it subsumed?
 C is subsumed by *D* (in symbols *C* ⊆ *D*) if we have for all interpretations C^I ⊂ D^I.
- Is *a* an instance of a concept *C*? *a* is an instance of *C* if for all interpretations, we have $a^{\mathcal{I}} \in C^{\mathcal{I}}$.
- Note: These questions can be posed with or without a TBox that restricts the possible interpretations.

Introduction

Concepts and Roles

TBox and ABox

Reasoning Services

Outlook

Literature

5 Outlook





- Can we reduce the reasoning services to perhaps just one problem?
- What could be reasoning algorithms?
- What can we say about complexity and decidability?
- What has all that to do with modal logics?
- How can one build efficient systems?

Concepts and Roles

TBox and ABox

Reasoning Services

Outlook

Literature



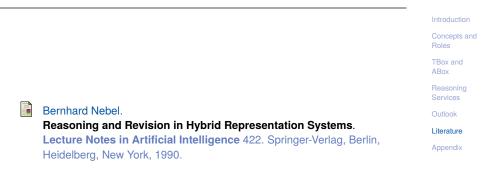
Literature I

| Baader, F., D. Calvanese, D. L. McGuinness, D. Nardi, and P. F. | Concepts and Roles |
|--|-----------------------|
| Patel-Schneider. The Description Logic Handbook: Theory, Implementation, | TBox and ABox |
| Applications, Cambridge University Press, Cambridge, UK, 2003. | Reasoning Services |
| Develop I. Development James O. Osharalar | Outlook |
| Ronald J. Brachman and James G. Schmolze. An overview of the KL-ONE knowledge representation system. | Literature |
| Cognitive Science, 9(2):171–216, April 1985. | Appendix |
| Franz Baader, Hans-Jürgen Bürckert, Jochen Heinsohn, Bernhard Hollunder, Jürgen Müller, Bernhard Nebel, Werner Nutt, and Hans-Jürgen Profitlich. | |
| Terminological Knowledge Representation: A proposal for a | |
| terminological logic. | |
| Published in Proc. International Workshop on Terminological Logics, | |
| 1991, DFKI Document D-91-13. | URG |

5

LL.

Literature II





Summary: Concept descriptions

| Abstract | Concrete | Interpretation | | Introduction |
|-----------------------------|--------------------------|---|---------|-----------------------|
| A | Α | $A^{\mathcal{I}}$ | | Concepts and |
| $C \sqcap D$ | (and <i>C D</i>) | $\mathcal{C}^{\mathcal{I}} \cap \mathcal{D}^{\mathcal{I}}$ | | Roles |
| $C \sqcup D$ | (or <i>C D</i>) | $\mathcal{C}^{\mathcal{I}} \cup \mathcal{D}^{\mathcal{I}}$ | | TBox and ABox |
| $\neg C$ | (not <i>C</i>) | $\mathcal{D} - \mathcal{C}^{\mathcal{I}}$ | | |
| $\forall r.C$ | (all <i>r C</i>) | $\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \subseteq oldsymbol{C}^\mathcal{I} ight\}$ | | Reasoning Services |
| $\exists r$ | (some r) | $\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}^{\mathcal{I}}(\boldsymbol{d}) \neq \boldsymbol{\emptyset} ight\}^{-1}$ | | Outlook |
| \geq n r | (atleast n r) | $\left\{ d \in \mathcal{D} : r^{\mathcal{I}}(d) \geq n \right\}$ | | Literature |
| \leq n r | (atmost <i>n r</i>) | $\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}_{-}^{\mathcal{I}}(\boldsymbol{d}) \leq \underline{n} \right\}$ | | Appendix |
| ∃r.C | (some r C) | $\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}^{\mathcal{I}}(\boldsymbol{d}) \cap \boldsymbol{C}^{\mathcal{I}} \neq \boldsymbol{\emptyset} ight\}$ | | |
| \geq n r.C | (atleast n r C) | $\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \cap oldsymbol{C}^\mathcal{I} \geq n ight\}$ | | |
| \leq n r.C | (atmost <i>n r C</i>) | $\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \cap oldsymbol{C}^\mathcal{I} \leq n ight\}$ | | |
| $r \stackrel{\cdot}{=} s$ | (eq <i>r s</i>) | $\left\{ d \in \mathcal{D} : r^{\mathcal{I}}(d) = s^{\mathcal{I}}(d) \right\}$ | | |
| $r \neq s$ | (neq <i>r s</i>) | $\left\{ \boldsymbol{d} \in \mathcal{D} : \boldsymbol{r}^{\mathcal{I}}(\boldsymbol{d}) \neq \boldsymbol{s}^{\mathcal{I}}(\boldsymbol{d}) ight\}$ | | |
| $r \sqsubseteq s$ | (subset r s) | $\left\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{r}^\mathcal{I}(oldsymbol{d}) \subseteq oldsymbol{s}^\mathcal{I}(oldsymbol{d}) ight\}$ | | |
| $g\stackrel{\cdot}{=}h$ | (eq <i>g h</i>) | $ig\{ oldsymbol{d} \in \mathcal{D} : oldsymbol{g}^\mathcal{I}(oldsymbol{d}) = oldsymbol{h}^\mathcal{I}(oldsymbol{d}) eq \emptyset ig\}$ | | 13 |
| g eq h | (neq <i>g h</i>) | $\left\{ d \in \mathcal{D} : \emptyset \neq g^{\mathcal{I}}(d) \neq h^{\mathcal{I}}(d) \neq \emptyset \right\}$ | | NKC N |
| $\{i_1, i_2, \ldots, i_n\}$ | (oneof $i_1 \dots i_n$) | $\{i_1^{\mathcal{I}}, i_2^{\mathcal{I}}, \dots, i_n^{\mathcal{I}}\}$ | r | |
| ember 23, 2015 | Nebel, Lindn | er, Engesser – KR&R | 35 / 36 | N |

November 23, 2015

Summary: Role descriptions

| | | | Concept Roles | s a |
|---------------------|-------------------|--|------------------|-----|
| | | | TBox an ABox | d |
| Abstract | Concrete | Interpretation | Reasoni | na |
| t | t | $t^{\mathcal{I}}$ | Services | |
| f | f | $f^{\mathcal{I}},$ (functional role) | Outlook | |
| <i>r</i> ⊓ <i>s</i> | (and <i>r s</i>) | $r^{\mathcal{I}} \cap s^{\mathcal{I}}$ | Literatur | е |
| r⊔s | (or <i>r s</i>) | $r^{\mathcal{I}} \cup s^{\mathcal{I}}$ | Appendi | v |
| $\neg r$ | (not <i>r</i>) | $\mathcal{D} 	imes \mathcal{D} - r^{\mathcal{I}}$ | Appendi | Ŷ |
| r ⁻¹ | (inverse r) | $egin{array}{llllllllllllllllllllllllllllllllllll$ | | |
| $r _C$ | (restr r C) | | | |
| r ⁺ | (trans r) | $(r^{\mathcal{I}})^+$ | | |
| <i>r</i> ∘ <i>s</i> | (compose r s) | $r^{\mathcal{I}} \circ s^{\mathcal{I}}$ | | |
| 1 | self | $\{(d,d): d \in \mathcal{D}\}$ | | |



and