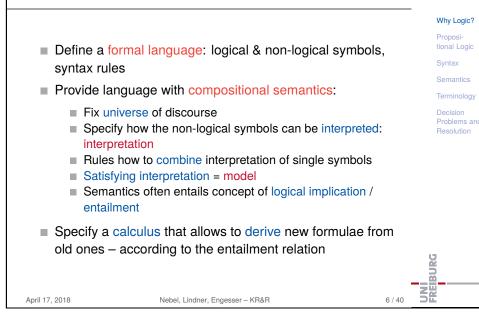
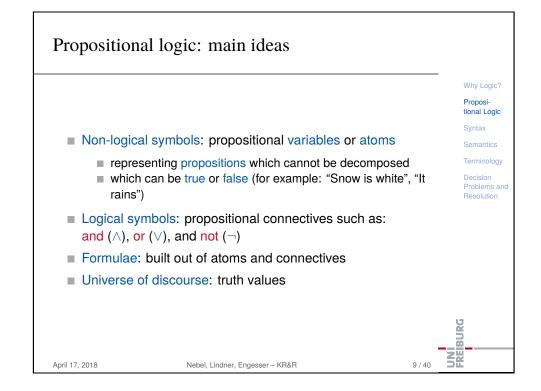
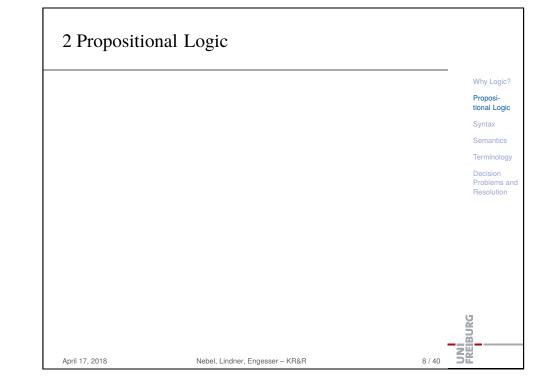
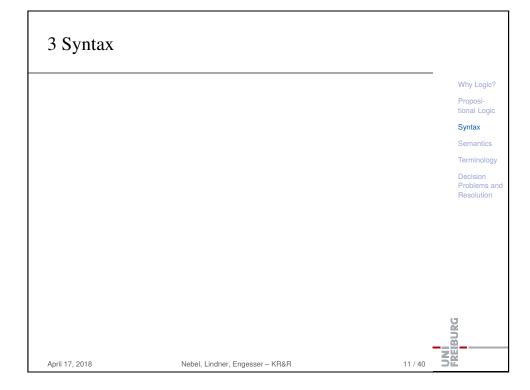


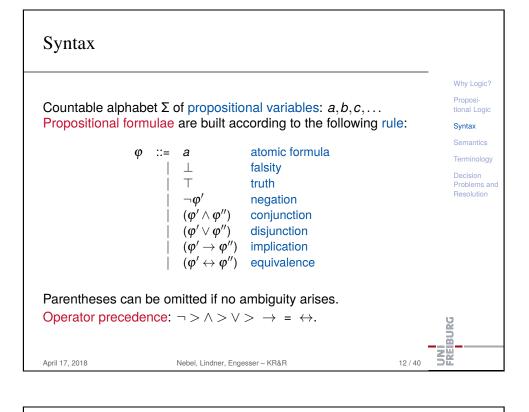
The logical approach

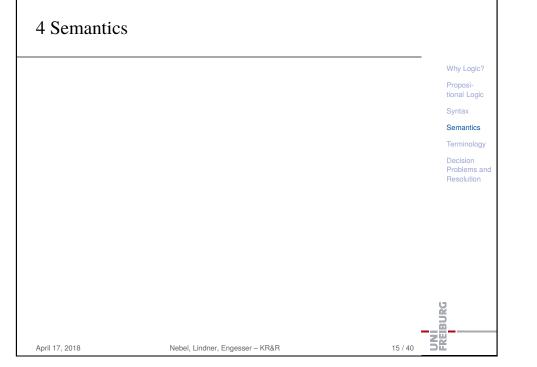




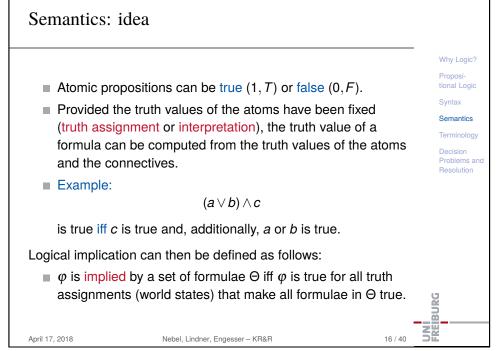


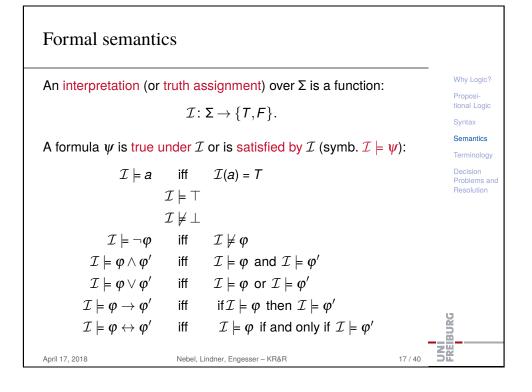


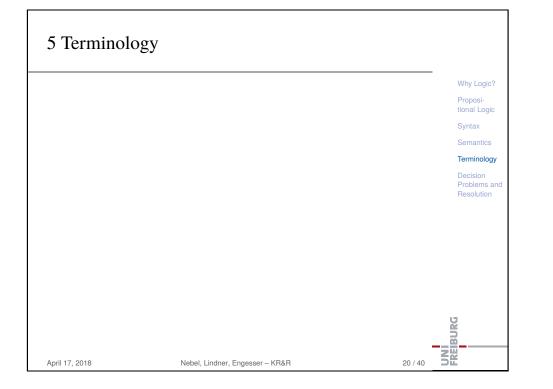




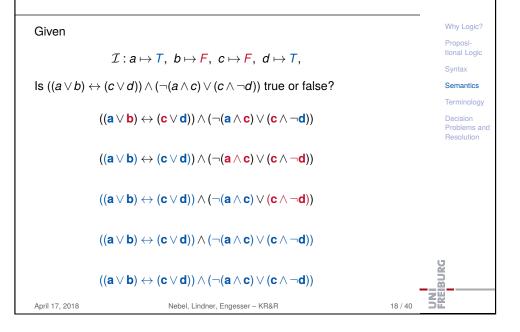
			Why Logic?
			Proposi- tional Logic
$(a \lor b)$ is a	an expression of the language of pro	positional	Syntax
logic.	in expression of the language of pro	poolional	Semantics
-			Terminology
in the lang	$ (\varphi' \leftrightarrow \varphi'') $ is a statement about hor juage of propositional logic can be for ng meta-language.	-	Decision Problems ar Resolution
	describe how expressions (in this c med, we use meta-language.	ase formulae)	
	describe how to interpret formulae, vulae, vulae	we use	
		-	
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Example



Terminolog	У						
			Why Logic?				
An interpretatio	n \mathcal{I} is a model of φ iff $\mathcal{I} \models \varphi$.		Proposi- tional Logic				
A formula φ is	, , ,		Syntax				
•	if there is an \mathcal{T} such that $\mathcal{T} \vdash \alpha$:		Semantics				
	satisfiable if there is an \mathcal{I} such that $\mathcal{I} \models \varphi$;						
unsatisfiab		Decision Problems and					
valid if $\mathcal{I} \models$		Resolution					
 falsifiable, 	otherwise.						
Formulae φ and all interpretation	d ψ are logically equivalent (symb. ϕ ns $\mathcal{I},$	$\equiv \psi$) if for					
	$\mathcal{I} \models \varphi$ iff $\mathcal{I} \models \psi$.						
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Examples

Satisfiable, unsatisfiable, falsifiable, valid? $(a \lor b \lor \neg c) \land (\neg a \lor \neg b \lor d) \land (\neg a \lor b \lor \neg d)$ \rightsquigarrow satisfiable: $a \mapsto T, b \mapsto F, d \mapsto F, ...$ \rightsquigarrow falsifiable: $a \mapsto F, b \mapsto F, c \mapsto T, ...$ $((\neg a \rightarrow \neg b) \rightarrow (b \rightarrow a))$ \rightsquigarrow satisfiable: $a \mapsto T, b \mapsto T$

 $\rightsquigarrow\,$ valid: Consider all interpretations or argue about falsifying ones.

Why Logic?

Propositional Logic

Syntax Semantics

Terminology

Decision Problems and Resolution

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Equivalence? $\neg(a \lor b) \equiv \neg a \land \neg b$

→ Of course, equivalent (de Morgan).

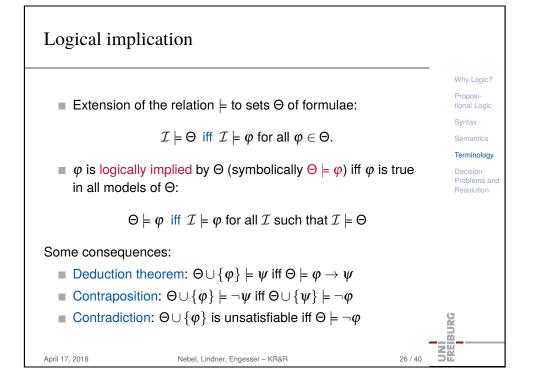
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Some equiv	valences							
							• Why Lo	ogic?
simplifications	$oldsymbol{arphi} ightarrow oldsymbol{\psi}$	≡	$ eg \phi \lor \psi$	$arphi \leftrightarrow \psi$	≡	$(arphi ightarrow \psi) \wedge$	Propos tional L	
						$(\psi ightarrow arphi)$	Syntax	
idempotency	$oldsymbol{arphi} ee oldsymbol{arphi}$	\equiv	φ	$oldsymbol{arphi}\wedgeoldsymbol{arphi}$	\equiv	ϕ	Seman	ntics
commutativity	$oldsymbol{arphi} ee oldsymbol{\psi}$			$oldsymbol{arphi}\wedgeoldsymbol{\psi}$		$oldsymbol{\psi} \wedge oldsymbol{arphi}$	Termin	ology
associativity			$\varphi \lor (\psi \lor \chi)$			$\varphi \wedge (\psi \wedge \chi)$	Decisio	on
absorption	$\varphi \lor (\varphi \land \psi)$		φ	$\varphi \land (\varphi \lor \psi)$		1	Probler Resolu	ms and
distributivity	$\varphi \wedge (\psi \lor \chi)$	≡	$(\varphi \land \psi) \lor$	$\varphi \lor (\psi \land \chi)$	≡	$(\varphi \lor \psi) \land$	nesolu	lion
double negation		≡	$(\phi \wedge \chi)$			$(\phi \lor \chi)$		
constants	$\neg \neg \phi$ $\neg \top$		ϕ	-	=	т		
De Morgan			$\neg \phi \land \neg \psi$		_	$\neg \varphi \lor \neg \psi$		
truth	$\varphi \lor \top$			$\varphi \wedge \top$				
falsity	$\varphi' \lor \bot$			$\varphi \wedge \bot$		•		
taut./contrad.	$\varphi \lor \neg \varphi$			$\phi \wedge \neg \phi$	\equiv	\perp		
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			Why I
Proposition			Propo tional
			Synta
• •	is unsatisfiable.		Sema
ϕ is satisfiable	iff $\neg \phi$ is falsifiable.		Termi
Proposition $\varphi \equiv \psi$ iff $\varphi \leftrightarrow \psi$	w is valid.		Decis Probl Reso
Theorem			
If $\varphi \equiv \psi$, and $\chi' \equiv \chi$.	χ' results from substituting $arphi$ by ψ in $\chi,$ the	n	
$\alpha d = \alpha d$			

How many	different formulae are there .				
			Why Logic?		
(alte state de la 50		Proposi- tional Logic		
for a given fi	for a given finite alphabet Σ ?				
Infinitely n	hany: $a, a \lor a, a \land a, a \lor a \lor a, \ldots$		Semantics		
How many	v different logically distinguishable (no	ot equivalent)	Terminology		
formulae?					
(if two	nula can be characterized by its set of m o formulae are not logically equivalent, th dels differ).		Resolution		
For Σ	with $n = \Sigma $, there are 2^n different interpr	retations.			
There	are $2^{(2^n)}$ different sets of interpretations				
There	are 2 ^(2ⁿ) (logical) equivalence classes o	of formulae.			
		_	BURG		
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Negation normal form

Theorem

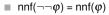
For each propositional formula there is a logically equivalent formula in NNF.

Proof.

First eliminate \rightarrow and \leftrightarrow by the appropriate equivalences. Base case: Claim is true for $a, \neg a, \top, \bot$. Inductive case: Assume claim is true for all formulae φ (up to a certain number of connectives) and call its NNF nnf(φ).

 $\blacksquare \ \operatorname{nnf}(\phi \land \psi) = (\operatorname{nnf}(\phi) \land \operatorname{nnf}(\psi))$

- $\quad \quad \mathsf{nnf}(\varphi \lor \psi) = (\mathsf{nnf}(\varphi) \lor \mathsf{nnf}(\psi))$
- $\blacksquare \operatorname{nnf}(\neg(\phi \land \psi)) = (\operatorname{nnf}(\neg\phi) \lor \operatorname{nnf}(\neg\psi))$
- $\blacksquare \operatorname{nnf}(\neg(\phi \lor \psi)) = (\operatorname{nnf}(\neg \phi) \land \operatorname{nnf}(\neg \psi))$



Normal forms	
	Why Logic?
Terminology:	Proposi- tional Logic
Atomic formulae a , negated atomic formulae $\neg a$, truth \top	Syntax
and falsity \perp are literals.	Semantics
A disjunction of literals is a clause.	Terminology
■ If ¬ only occurs in front of an atom and there are no → and ↔, the formula is in negation normal form (NNF). Example: $(\neg a \lor \neg b) \land c$, but not: $\neg (a \land b) \land c$	Decision Problems and Resolution
 A conjunction of clauses is in conjunctive normal form (CNF). 	
Example: $(a \lor b) \land (\neg a \lor c)$	
 The dual form (disjunction of conjunctions of literals) is in disjunctive normal form (DNF). Example: (a ∧ b) ∨ (¬a ∧ c) 	BURG

Theorem

Why Logic?

tional Logic

Semantics

Terminology

Problems and

Resolution

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Decision

Proposi-

Syntax

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For each propositional formula there exist logically equivalent formulae in CNF and DNF, respectively.

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Proof.

The claim is true for $a, \neg a, \top, \bot$.

Conjunctive normal form

Let us assume it is true for all formulae φ (up to a certain number of connectives) and call its CNF $cnf(\varphi)$ (and its DNF $dnf(\varphi)$).

- $cnf(\neg \phi) = nnf(\neg dnf(\phi))$ and $cnf(\phi \land \psi) = cnf(\phi) \land cnf(\psi)$.
- Assume $\operatorname{cnf}(\varphi) = \bigwedge_i \chi_i$ and $\operatorname{cnf}(\psi) = \bigwedge_j \rho_j$ with χ_i, ρ_j being clauses. Then $\operatorname{cnf}(\varphi \lor \psi) = \operatorname{cnf}((\bigwedge_i \chi_i) \lor (\bigwedge_i \rho_i)) = \bigwedge_i \bigwedge_i (\chi_i \lor \rho_i)$ (by distributivity)

Similar for $dnf(\phi)$.

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Why Logic?

tional Logic

Semantics

Terminology

Problems and

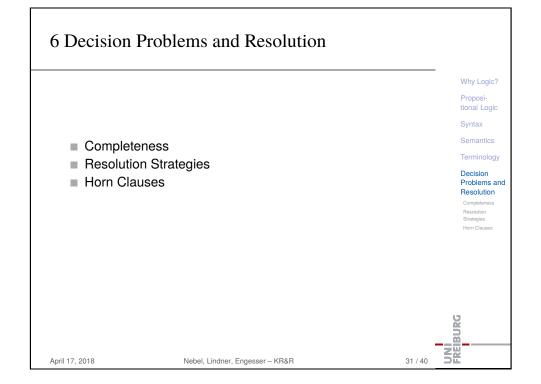
Resolution

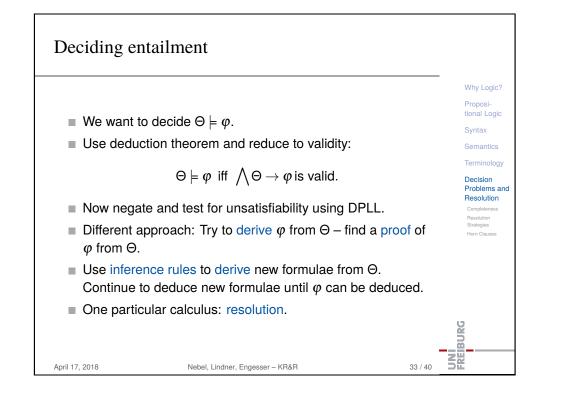
Decision

Proposi-

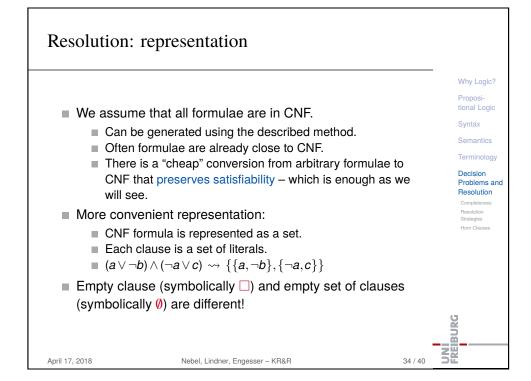
Syntax

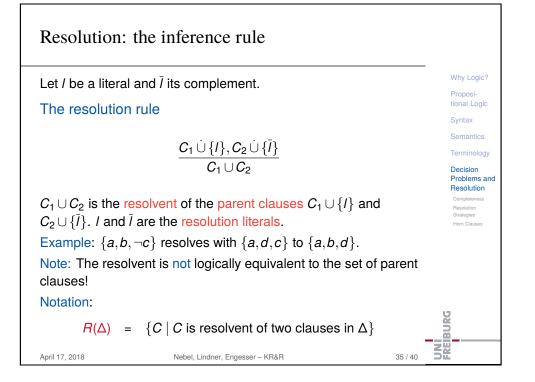
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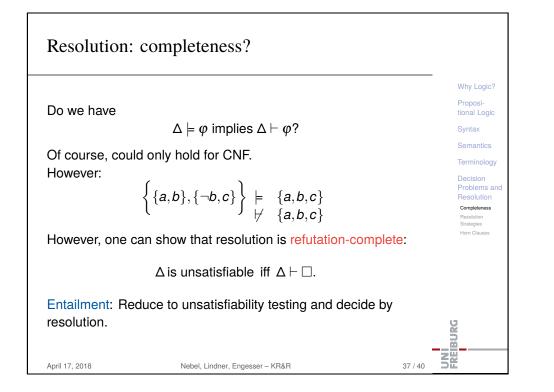




How to decide properties of formulae Why Logic? How do we decide whether a formula is satisfiable, unsatisfiable, Proposivalid, or falsifiable? tional Logic Syntax Note: Satisfiability and falsifiability are NP-complete. Validity and unsatisfiability are co-NP-complete. A CNF formula is valid iff all clauses contain two Decision Problems and complementary literals or \top . Resolution A DNF formula is satisfiable iff one disjunct does not Resolution contain \perp or two complementary literals. Horn Clause However, transformation to CNF or DNF may take exponential time (and space!). One can try out all truth assignments. One can test systematically for satisfying truth assignments BURG (backtracking) ~> Davis-Putnam-Logemann-Loveland. **NU** 32 / 40 April 17, 2018 Nebel, Lindner, Engesser - KR&R

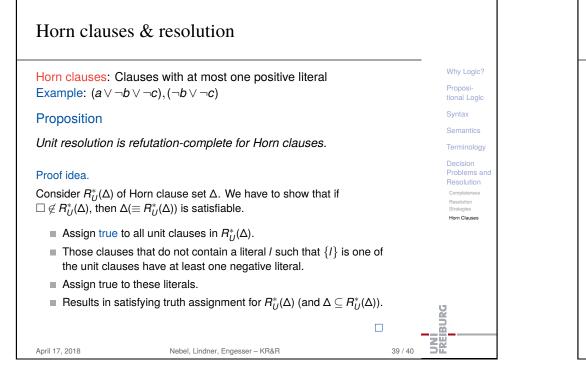






<i>D</i> can be derived f there is a sequence	Why Logic? Proposi- tional Logic		
•	$C_i \in R(\Delta \cup \{C_1, \dots, C_{i-1}\}), \text{ for all } i \in \{C_i \in C_i\}$	้1 กไ	Syntax
		<u>,</u> ,, <i>II</i> }.	Semantics
Define $R^*(\Delta) = \{D\}$	$ \Delta \vdash D \}.$		Terminology
Theorem (Sound	Iness of resolution)		Decision Problems and
Let D be a clause.	If $\Delta \vdash D$ then $\Delta \models D$.		Completeness Resolution Strategies
Proof idea.			Horn Clauses
Let $C_1 \cup \{I\}$ and $C_2 \cup \{I\}$ Assume $\mathcal{I} \models \Delta$, we have	$p \in C_2$ s.t. $\mathcal{I} \models m$. This implies $\mathcal{I} \models D$.		
	model \mathcal{I} of Δ also satisfies D , i.e., $\Delta \models D$.		JURG
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Resolution s	trategies		
			Why Logic?
			Proposi- tional Logic
Trying out	all different resolutions can be very costl	ly,	Syntax
and might	not be necessary.		Semantics
There are a	different resolution strategies.		Terminology
 Examples: 	and characteristic and constrained in the strategies.		Decision Problems and Resolution
parent Unit re parent Not all	esolution ($R_I(\cdot)$): In each resolution step, on clauses must be a clause of the input set. solution ($R_U(\cdot)$): In each resolution step, one clauses must be a unit clause. strategies are (refutation) completeness pre- r input nor unit resolution is. However, there	e of the eserving.	Completeness Resolution Strategies Horn Clauses
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Einführung in	aus, J. Flum, and W. Thomas. n die mathematische Logik. iche Buchgesellschaft, Darmstadt, 1986.		Resolution Strategies Horn Clauses
U. Schöning. Logik für Info Spektrum-Verl	ormatiker. lag, 5th edition, 2000.		
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