Principles of Knowledge Representation and Reasoning

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Exercise Sheet 8

Due: June 21th, 2018

Exercise 8.1 (Expressibility and Complexity, 3 + 3)

(a) You are asked to show that concept constructors are not just syntactic sugar. In particular, if one adds the functionality concept constructor — $(\leq 1 r)$ — to $\mathcal{ALC}$ one gets the language $\mathcal{ALCF}$. Show that $\mathcal{ALCF}$ is indeed more expressive than $\mathcal{ALC}$ by proving that the concept $(\leq 1 r)$ cannot be expressed in $\mathcal{ALC}$.
Hint: Assume an $\mathcal{ALC}$ concept C being equivalent to $(\leq 1r)$. Provide an interpretation $\mathcal{I}$ which satisfies both C and $(\leq 1r)$. Then construct another interpretation $\mathcal{I}'$ from $\mathcal{I}$ as follows: $\Delta^{\mathcal{I}'} = \Delta^{\mathcal{I}} \times \mathbb{N}, A^{\mathcal{I}'} = \{(d,i) | d \in A^{\mathcal{I}}, i \in \mathbb{N})\}, r^{\mathcal{I}'} = \{(< d, i >, < e, j > | (d,e) \in r^{\mathcal{I}}, i, j \in \mathbb{N}))$ and show that $\mathcal{I}'$ is a model for C but not for $(\leq 1r)$.

(b) Given the TBox $\mathcal{T}$, determine the description logic used to describe the concepts. What is the complexity class of the satisfiability problem of this logic? Propose how $\mathcal{T}$ could be expressed in a less complex DL.

- $\mathcal{T} = \{A \doteq \exists r.(\forall s.C \sqcup \exists s.\neg C), B \doteq (\geq 1s), C \doteq B \sqcap \forall r.\neg(\neg A \sqcup \neg B)$

Exercise 8.2 (Nonmonotonic Reasoning using Abnormality Predicates, 3)
Consider the following knowledge base KB and show that $KB \models_{\leq} flies(c) \vee flies(d)$. *Note:* The special version of entailment (which is called *minimal entailment* and which is denoted by $\models_{\leq}$) is defined as follows: $KB \models_{\leq} \phi$ holds iff for every interpretation $\mathcal{I}$ such that $\mathcal{I} \models KB$, either $\mathcal{I} \models \phi$ or there is an $\mathcal{I}'$ such that $\mathcal{I}' < \mathcal{I}$ and $\mathcal{I}' \models KB$ (with $\mathcal{I}' \leq \mathcal{I}$ iff $Abnormal^{\mathcal{I}'} \subseteq Abnormal^{\mathcal{I}}$). Discuss how this kind of reasoning compares to reasoning under the Closed World Assumption.

$$KB = \{\forall x(Bird(x) \wedge \neg Abnormal(x) \rightarrow flies(x)), Bird(c), Bird(d), \neg flies(c) \vee \neg flies(d)\}$$

Exercise 8.3 (Extensions in Default Logic, 3)
Consider the propositional default theory $\Delta = \langle D, W \rangle$ with

$$D = \{\frac{\top : m}{m}, \frac{\top : i}{i}, \frac{m : \neg s}{\neg s}, \frac{m : b}{b}, \frac{i : s \wedge \neg b}{s \wedge \neg b}\}, W = \{\neg(m \wedge i)\}$$

Determine all extensions of Δ. Which of the propositions s, b, $s \vee b$, $s \wedge b$ are entailed by Δ using credulous reasoning? Which of them are entailed using skeptical reasoning?