Principles of Knowledge Representation and Reasoning

B. Nebel, F. Lindner, T. Engesser — University of Freiburg
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Exercise Sheet 7

Due: June 14th, 2018

Exercise 7.1 (Unfolding, 3)

Specify the unfolding of the concepts `Granddaughter`, `Mother-without-daughter`, and `Bigamist` wrt. the TBox in exercise 5.2(a). Determine the primitive components used in your definitions. Provide an initial interpretation: use the ABox given in the lecture (chapter 7, slide 26). Finally, specify the full interpretation of these three concepts as induced by your initial interpretation.

Exercise 7.2 (Structural Subsumption Algorithm, 3)

Given the $\mathcal{FL}^-$ concepts C' and D' and terminology $\mathcal{T}$ you are asked to use the structural subsumption algorithm from the lecture to prove or disprove $\mathcal{T} \models C' \sqsubseteq D'$. You will need to apply normalization and unfolding as preprocessing steps.

- $\mathcal{T} = \{A \sqsubseteq \forall r_1.(\exists r_2 \sqcap C), B \sqsubseteq \forall r_1.(\exists r_3 \sqcap \forall r_2.D), D \equiv \exists r_2 \sqcap \forall r_2.C\}$
- $C' \equiv A \sqcap B$
- $D' \equiv \forall r_1.(\forall r_2.\exists r_2) \sqcap A$

Exercise 7.3 (Tableau Algorithm and Reasoning Services, 2 + 2 + 2)

In this exercise you are asked to apply the tableau algorithm for description logics (lecture 9) and to use it to answer questions about TBoxes and ABoxes.[1]

(a) Use the tableau algorithm to show that the $\mathcal{ALC}$ concept $C \doteq \forall r.(\neg A \sqcup \exists s.A) \sqcap \exists r.(A \sqcap \exists s.\neg A)$ is satisfiable. Extract a model of C from your tableau.

(b) Given two $\mathcal{ALCQ}$ concepts A and B, use tableau to prove that $A \sqsubseteq B$.
Hint: To transform a concept description that contains (qualified) number restrictions to negation normal form, you may need to make use of the following equivalences: $\neg(\geq n + 1r.C) \equiv (\leq nr.C), \neg(\geq 0r.C) \equiv \bot, \neg(\leq nr.C) \equiv (\geq n + 1r.C)$. The rule for expanding (qualified) number restrictions is explained in the Figure 2.6 mentioned in the footnote. Moreover, one needs to extend the clash detection, viz., there is a clash in a branch $\mathcal{B}$ if $\{(\leq nR(x))\} \cup \{R(x, y_i)|1 \leq i \leq n + 1\} \cup \{y_i \neq y_j|1 \leq i < j \leq n + 1\} \subseteq \mathcal{B}$ for individual names $x, y_1, \ldots, y_{n+1}$, a nonnegative integer n, and a role name R.[2]

- $A \doteq \exists r.(\leq 2r.C) \sqcap \forall r.C$
- $B \doteq \forall r.(C \sqcup D) \sqcap \exists r.(\leq 3r.C)$

(c) Explain how one can you use the tableau procedure to retrieve all instances of the concept C' given the ABox $\mathcal{A}$. Exemplify your approach by choosing any of the individuals and prove or disprove that it is an instance of C'.

- $\mathcal{A} = \{A(a), A(c), B(b), \neg C(d), r(b, c), r(c, d), s(a, b), s(a, c), s(c, c)\}$
- $C' \doteq \exists s.B \sqcup \exists s.\exists r.\neg C$

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[1] You may also want to consult `https://www.inf.unibz.it/~franconi/dl/course/dlhb/dlhb-02.pdf` (p. 85, Fig. 2.6) for an overview of some basic tableau expansion rules.

[2] Cf., `https://www.inf.unibz.it/~franconi/dl/course/dlhb/dlhb-02.pdf`, page 86