Principles of Knowledge Representation and Reasoning

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Summer Semester 2018

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Exercise Sheet 3

Due: May 10th, 2018

Exercise 3.1 (Modeling Belief, 2+2)

You are asked to use Kripke frames to model the beliefs of three players (Alice, Bob, and yourself) playing a game. The game is called *aces and eights*. The deck consists of four aces and four eights. Each of the players receives two cards. Without looking at the cards, each of the players raises them up to his/her forehead such that the other players can see them but he/she cannot. Now, each of the players' task it to determine what cards he/she is holding. (The suits of the cards do not matter.)

(a) Suppose that Alice holds two aces and Bob holds two eights. Model the hypotheses about the cards you may be holding as possible worlds of a Kripke frame. Use an accessibility relation R_{me} to represent that two possible worlds are indistinguishable from your perspective.

(b) Suppose that Alice and Bob announce that they cannot determine what cards they are holding. Does this help you to figure out what cards you are holding? To answer this question extend your Kripke frame representation to include also the perspectives of Alice and Bob on the game situation. You will need to add possible worlds and two accessibility relations, R_{Alice} and R_{Bob}, to represent which possible worlds are indistinguishable from Alice's and Bob's perspectives.

Exercise 3.2 (Modal Logic: Tableaux Rules, 2+2+2)

Use tableaux to prove or disproof the following statements. Construct a counterexample from the tableau when possible.

(a) $\Box\Diamond p \leftrightarrow \Box\Diamond\Box\Diamond p$ is S4-valid.

(b) $\Box(\Box p \rightarrow q) \vee \Box(\Box q \rightarrow p)$ is S4-valid.

(c) $\Diamond(p \wedge \Box q) \rightarrow \Box(p \vee \Diamond q)$ is KT5-valid.

Exercise 3.3 (Characteristic Axioms, 2)

Proof the following statement: A frame $\mathcal{F}$ is transitive if and only if $\mathcal{F} \models \Box\phi \rightarrow \Box\Box\phi$ for each modal formula ϕ.