

# Game Theory

## 1. Introduction

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# What is Game Theory?

What is  
Game  
Theory?

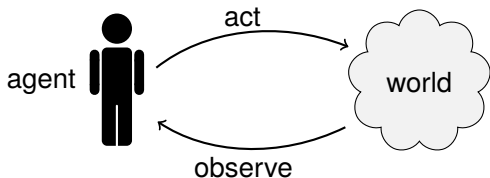
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Consider **rationally acting agents**:



Rational agents maximize their (expected) utility:

- decision theory
- Markov decision processes (MDPs)
- reinforcement learning
- AI planning
- ...

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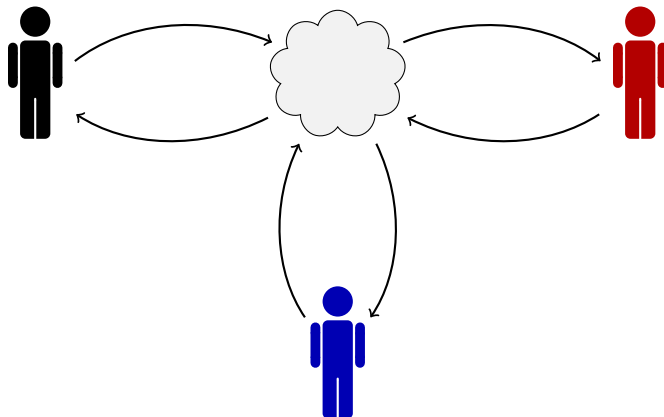
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Situation in **game theory**:



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Multiple rational agents interacting in **strategic decision situations**.

- resulting utility depends on what other agents do.
- all agents know that other agents are **rational** (this is even common knowledge).

**Interesting questions:**

- how to **model** such strategic situations
- how to **solve** such strategic situations
- how to **design games** that have desired solutions

Game theory is the study and analysis of such strategic decision situations.

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- originally part of **mathematics** and theoretical **economics**
- today **ubiquitous**
- here: **artificial intelligence** and **computer science** perspective
  - **rationality assumptions** (“homo economicus”) more warranted for artificial agents than for humans
  - interesting **algorithmic** questions

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# Application Examples

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## Two-player board and card games:

- very special
- whatever is good for one player is bad for the other (**strictly competitive** games)
- recent visible success: Poker (no-limit, heads-up, hold'em)

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**Auctions:** Think of eBay, Google AdWords, ...

- **setting:** one object should be allocated to one out of a number of bidders.
- **questions:**
  - what bidding **protocol** to use?
  - who is the **winner**?
  - what does the winning bidder have to **pay**?

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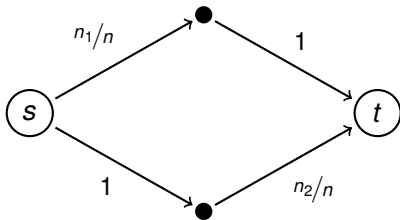
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**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



**Question:** Assume that there are  $n = 2$  agents.  
Which routes will they choose?

**Average travel cost per agent:** ?

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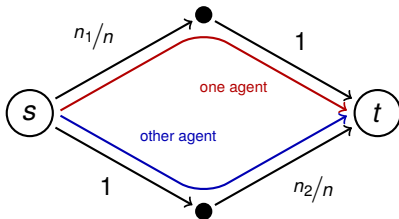
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**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



**Question:** Assume that there are  $n = 2$  agents.  
Which routes will they choose?

**Average travel cost per agent:** 1.5

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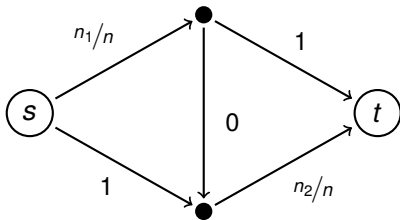
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**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



**Question:** Assume that there are  $n = 2$  agents.

Which routes will they choose **now (with free new road)**?

**Average travel cost per agent: ?**

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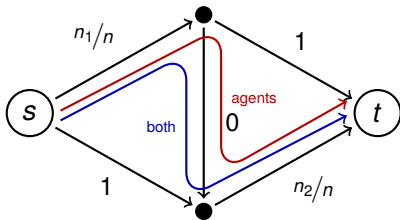
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**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



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**Question:** Assume that there are  $n = 2$  agents.

Which routes will they choose **now (with free new road)**?

**Average travel cost per agent:**  $2 > 1.5$



## Security games:

- **setting:** a facility (e. g., an airport) has to be guarded to avoid attacks
- **possible methods:**
  - visit all critical places
  - choose the places probabilistically
  - find a probability distribution for the routing that minimizes expected damage even under the assumption that the attacker can observe the guards

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- **setting:** a set of alternatives (candidates) and a set of voters, determine winner or ranking
- **questions:**
  - what questions to ask?
  - how to determine a winner / ranking?
  - what is the computational complexity of determining a winner?
  - can the protocol be made manipulation-safe?

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# Rationality

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## Rationality:

- **General assumption:** All players want to maximize their own utility and nothing else.
- **Contrasts:**
  - **Altruistic** agents want to maximize utility of other agents
  - **Cooperative** agents want to maximize group utility
  - **Byzantine** agents want to **minimize** utility of other agents

## Limitations:

- agents may not foresee all consequences of their decisions (**bounded rationality**)
- agents may not know all relevant information about the game structure (**incomplete information**)
- agents may not know all relevant information about the current state of the game (**imperfect information**)

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# Course Outline

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- strategic games
- extensive games
- repeated games
- imperfect information games and Poker
- social choice theory
- mechanism design

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# Let's Play a Game

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We play a game called “Beauty Contest”.

## Rules

Everybody chooses a natural number  $n$  with  $1 \leq n \leq 100$ .  
The players that come closest to  $\frac{2}{3}$  of the average win.

Now it's your turn!

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