

# Game Theory

## 1. Introduction

Albert-Ludwigs-Universität Freiburg



**UNI  
FREIBURG**

Bernhard Nebel and Robert Mattmüller

Summer semester 2018

# 1 What is Game Theory?



What is  
Game  
Theory?

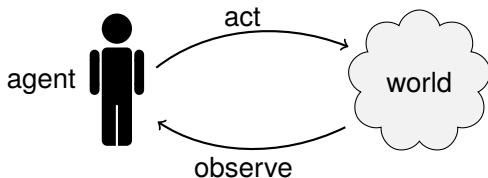
Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

Consider **rationaly acting agents**:



Rational agents maximize their (expected) utility:

- decision theory
- Markov decision processes (MDPs)
- reinforcement learning
- AI planning
- ...

What is  
Game  
Theory?

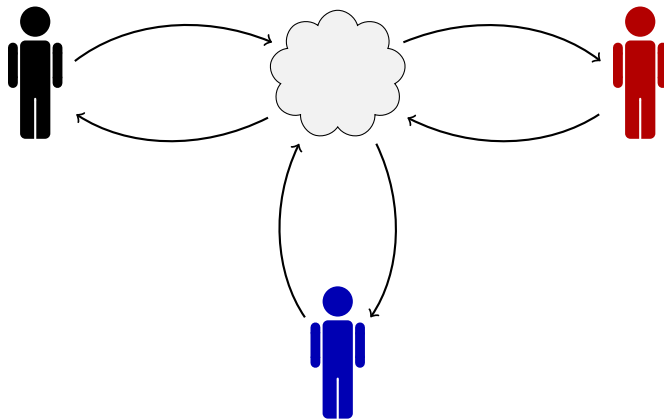
Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

Situation in **game theory**:



What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

Multiple rational agents interacting in **strategic decision situations**.

- resulting utility depends on what other agents do.
- all agents know that other agents are **rational** (this is even common knowledge).

Interesting questions:

- how to **model** such strategic situations
- how to **solve** such strategic situations
- how to **design games** that have desired solutions

Game theory is the study and analysis of such strategic decision situations.

What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

- originally part of **mathematics** and theoretical **economics**
- today **ubiquitous**
- here: **artificial intelligence** and **computer science** perspective
  - **rationality assumptions** (“homo economicus”) more warranted for artificial agents than for humans
  - interesting **algorithmic** questions

What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

## 2 Application Examples



What is  
Game  
Theory?

**Application  
Examples**

Rationality

Course  
Outline

Let's Play a  
Game

## Two-player board and card games:

- very special
- whatever is good for one player is bad for the other (**strictly competitive** games)
- recent visible success: Poker (no-limit, heads-up, hold'em)

What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game



**Auctions:** Think of eBay, Google AdWords, ...

- **setting:** one object should be allocated to one out of a number of bidders.
- **questions:**
  - what bidding **protocol** to use?
  - who is the **winner**?
  - what does the winning bidder have to **pay**?

What is  
Game  
Theory?

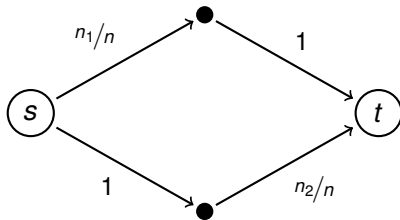
Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



**Question:** Assume that there are  $n = 2$  agents.  
Which routes will they choose?

**Average travel cost per agent: ?**

What is  
Game  
Theory?

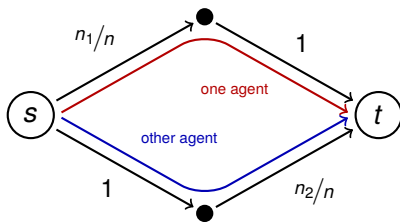
Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



**Question:** Assume that there are  $n = 2$  agents.  
Which routes will they choose?

**Average travel cost per agent:** 1.5

What is  
Game  
Theory?

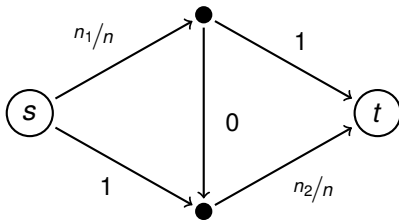
Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



**Question:** Assume that there are  $n = 2$  agents.

Which routes will they choose **now (with free new road)**?

**Average travel cost per agent: ?**

What is  
Game  
Theory?

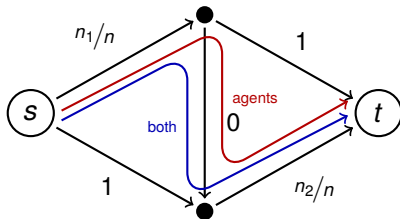
Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

**Congestion games:** road network with travel costs dependent on the number of agents choosing a particular road



What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

**Question:** Assume that there are  $n = 2$  agents.

Which routes will they choose **now (with free new road)**?

**Average travel cost per agent:**  $2 > 1.5$

## Security games:

- **setting:** a facility (e. g., an airport) has to be guarded to avoid attacks
- **possible methods:**
  - visit all critical places
  - choose the places probabilistically
  - find a probability distribution for the routing that minimizes expected damage even under the assumption that the attacker can observe the guards

What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

- **setting:** a set of alternatives (candidates) and a set of voters, determine winner or ranking
- **questions:**
  - what questions to ask?
  - how to determine a winner / ranking?
  - what is the computational complexity of determining a winner?
  - can the protocol be made manipulation-safe?

What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

# 3 Rationality



What is  
Game  
Theory?

Application  
Examples

**Rationality**

Course  
Outline

Let's Play a  
Game



## Rationality:

- **General assumption:** All players want to maximize their own utility and nothing else.
- **Contrasts:**
  - **Altruistic** agents want to maximize utility of other agents
  - **Cooperative** agents want to maximize group utility
  - **Byzantine** agents want to **minimize** utility of other agents

## Limitations:

- agents may not foresee all consequences of their decisions (**bounded rationality**)
- agents may not know all relevant information about the game structure (**incomplete information**)
- agents may not know all relevant information about the current state of the game (**imperfect information**)

What is  
Game  
Theory?

Application  
Examples

**Rationality**

Course  
Outline

Let's Play a  
Game

# 4 Course Outline



What is  
Game  
Theory?

Application  
Examples

Rationality

**Course  
Outline**

Let's Play a  
Game

- strategic games
- extensive games
- repeated games
- imperfect information games and Poker
- social choice theory
- mechanism design

What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game

# 5 Let's Play a Game



What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

**Let's Play a  
Game**

We play a game called “Beauty Contest”.

## Rules

Everybody chooses a natural number  $n$  with  $1 \leq n \leq 100$ .  
The players that come closest to  $\frac{2}{3}$  of the average win.

Now it's your turn!

What is  
Game  
Theory?

Application  
Examples

Rationality

Course  
Outline

Let's Play a  
Game