

## Introduction to Game Theory

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### Exercise Sheet 12

**Due: Monday, July 16, 2018**

#### Exercise 12.1 (Stable matchings, 2 points)

Apply the deferred acceptance algorithm with male proposals to the following problem and state what happens in the iterations:

- *Man 1:*  $w_4 \prec_{m_1} w_3 \prec_{m_1} w_1 \prec_{m_1} w_2$
- *Man 2:*  $w_3 \prec_{m_2} w_2 \prec_{m_2} w_1 \prec_{m_2} w_4$
- *Man 3:*  $w_4 \prec_{m_3} w_2 \prec_{m_3} w_3 \prec_{m_3} w_1$
- *Man 4:*  $w_4 \prec_{m_4} w_1 \prec_{m_4} w_3 \prec_{m_4} w_2$
- *Woman 1:*  $m_4 \prec_{w_1} m_2 \prec_{w_1} m_3 \prec_{w_1} m_1$
- *Woman 2:*  $m_2 \prec_{w_2} m_1 \prec_{w_2} m_4 \prec_{w_2} m_3$
- *Woman 3:*  $m_1 \prec_{w_3} m_3 \prec_{w_3} m_2 \prec_{w_3} m_4$
- *Woman 4:*  $m_4 \prec_{w_4} m_1 \prec_{w_4} m_2 \prec_{w_4} m_3$

Preferences are given from lowest (left) to highest (right).

#### Exercise 12.2 (Top trading cycle method, 2+2+2 points)

- Prove that the top trading cycle algorithm returns an allocation that is in the core.
- Prove that the top trading cycle mechanism cannot be manipulated.
- Apply the top trading cycle algorithm to the following problem and state what happens in the iterations:

- *Player 1:*  $1 \triangleleft_1 4 \triangleleft_1 2 \triangleleft_1 3$
- *Player 2:*  $3 \triangleleft_2 2 \triangleleft_2 1 \triangleleft_2 4$
- *Player 3:*  $2 \triangleleft_3 3 \triangleleft_3 4 \triangleleft_3 1$
- *Player 4:*  $2 \triangleleft_4 1 \triangleleft_4 4 \triangleleft_4 3$

Preferences are given from lowest (left) to highest (right).

The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.