Introduction to Game Theory

B. Nebel, R. Mattmüller — University of Freiburg
T. Schulte, D. Bergdoll — Department of Computer Science

Summer semester 2018

Exercise Sheet 11

Due: Monday, July 9, 2018

Exercise 11.1 (May's theorem, 3 points)

Recall May's theorem: A social choice function $f : L^n \to A$ for a set of two alternatives $A = \{x, y\}$ satisfies anonymity, neutrality and monotonicity iff it is the plurality method (i.e., $f(\prec_1, \dots, \prec_n) = x$ iff $\#\{i \mid y \prec_i x\} \geq \frac{n}{2}$).

We assume n is odd to avoid tie-breaking issues that could violate neutrality.

Show that each of the three conditions is necessary for May's theorem.

(a) anonymity, i.e., $f(\prec_1, \dots, \prec_n) = f(\prec_{\pi(1)}, \dots, \prec_{\pi(n)})$ for all permutations π of the voters $\{1, \dots, n\}$.

(b) neutrality, i.e., $f(\prec_1, \dots, \prec_n) = x$ iff $f(\prec'_1, \dots, \prec'_n) = y$, where $x \prec'_i y$ iff $y \prec_i x$ for all $i = 1, \dots, n$.

(c) monotonicity, i.e., if $f(\prec_1, \dots, \prec_n) = x$, then also $f(\prec'_1, \dots, \prec'_n) = x$, where $\prec'_i = \prec_i$ for $i \neq I$ for some voter I such that $x \prec_I y$ and $y \prec'_I x$.

Hint: For each condition, find a counterexample (a social choice function) that fulfills all other conditions but the one in question and that is not the plurality method.

Exercise 11.2 (Single peaked preferences, 1 + 2 points)

(a) Allan, Mark, and Kenneth discuss how much time to invest in collective preparations for their upcoming exam in game theory. Their valuations over the amount of time $x \in \mathbb{R}^{>0}$ (in hours) to invest are as follows:

$$v_{Allan}(x) = -\frac{7}{3} + \frac{7}{3}x - \frac{1}{15}x^2$$
$$v_{Mark}(x) = -\frac{1}{2}x + 20$$
$$v_{Kenneth}(x) = 4x - \frac{1}{5}x^2$$

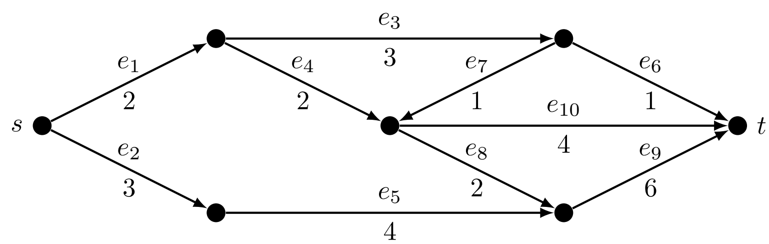
To agree on a fixed amount of time $x \in [5, 30]$, Allan, Mark, and Kenneth take a vote in which each of them submits a single peaked preference relation. On what amount of time will they agree using the median rule?

(b) Show that the median rule is not incentive compatible when the preference relations are not restricted to be single peaked. Construct valuation functions for Allan, Mark, and Kenneth, such that at least one of them has an incentive to misrepresent their true preferences.

Exercise 11.3 (Network routing as VCG-mechanism, 2 points)

Let $G = (V, E)$ be a directed graph. Every edge $e \in E$ belongs to a player e and generates cost c_e when being used. We want to rent a path between the two nodes s and t. The set of alternatives A contains all paths between s and t. Player e has cost c_e, if edge e lies on the chosen path p, zero otherwise. Maximization of social welfare means minimizing $\sum_{e \in p} c_e$ over all paths p from s to t.

Which alternatives does the VCG-mechanism choose in the following concrete example? Which payments result from this?
Please justify your answers.



The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.