Introduction to Game Theory

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Exercise Sheet 7

Due: Monday, June 11, 2018

Exercise 7.1 (Extensive games with simultaneous moves, 4 points)

There is a group of 1000 pirates, who are all extremely greedy, heartless, and rational. Also, every pirate knows that every other pirate has this attitude as well. Their respective position in the group is higher the earlier they joined the group, from pirate 1 down to pirate 1000. The pirates found a treasure and have to decide how to split it among themselves. Every day they vote whether to kill the lowest ranked pirate or to split the treasure among the living pirates. If at least 50% vote for splitting the treasure, they will do so. Otherwise, the lowest ranked pirate is killed and the procedure continues on the next day. When will the treasure be split up and how does the voting proceed?

Exercise 7.2 (Repeated Games, 4 points)

Consider the infinitely repeated prisoner's dilemma with discount factor $0 < \delta < 1$. The payoff matrix of the stage game is given below.

		Player 2	
		C	D
Player 1	C	$5, 5$	$0, 6$
	D	$6, 0$	$1, 1$

(a) Let t be the *tit-for-tat* strategy as defined in the lecture. Specify the unique run $O(t, t)$ that results from playing t against t.

(b) Compute the discounted payoff $v_1(O(t, t))$ of player 1 for the strategy profile (t, t) for general $0 < \delta < 1$ and for $\delta = \frac{1}{2}$ in particular.

(c) Assume that player 1 deviates from t and plays some other strategy s instead, whereas player 2 still plays t. Without loss of generality, assume further that the first deviation occurs in the first round, where player 1 defects (D) instead of cooperating (C).

Show that, for $\delta = \frac{1}{2}$, we have $v_1(O(s, t)) \leq v_1(O(t, t))$.

(d) Does the result of part (c) give us any Nash equilibria in the infinitely repeated prisoner's dilemma with discount factor $\delta = \frac{1}{2}$?

The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate all names on your solution.