Introduction to Game Theory

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Exercise Sheet 3

Due: Monday, May 7, 2018

Exercise 3.1 (Best response function, 3 points)

Let $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ with $N = \{1, 2\}$, $A_1 = A_2 = \mathbb{R}^{\geq 0}$, $u_1(a_1, a_2) = a_1(a_2 - a_1)$ and $u_2(a_1, a_2) = a_2(1 - \frac{1}{2}a_1 - a_2)$ for all $(a_1, a_2) \in A$.
Define all Nash equilibria of this game by constructing and analyzing the best response function of both players.

Exercise 3.2 (Kakutani's fixed point theorem, 1+1+1+1 points)

Let X be a compact, convex, non-empty subset of $\mathbb{R}^n$ and let $f : X \to 2^X$ be a set-valued function for which

- for each $x \in X$, the set $f(x)$ is nonempty and convex;
- the graph of f is closed (i.e. for all sequences $\{x_k\}$ and $\{y_k\}$ such that $y_k \in f(x_k)$ for all k, $x_k \to x$, and $y_k \to y$, we have $y \in f(x)$).

Then there exists $x^* \in X$ such that $x^* \in f(x^*)$.

Show that each of the following four conditions is necessary for Kakutani's theorem.

(a) X is compact.

(b) X is convex.

(c) $f(x)$ is convex for each $x \in X$.

(d) f has a closed graph.

Hint: There exists a counter-example with $n = 1$ in each of the four cases.

The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate both names on your solution.