## Introduction to Game Theory

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## Exercise Sheet 2 Due: Monday, April 30, 2018

**Exercise 2.1** (Elimination of strictly dominated strategies, 3+1 points) Consider the game  $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$  with  $N = \{1, 2\}, A_i = \{a_i, b_i, c_i, d_i\}, i = 1, 2$ , and the following payoff matrix.

		Player 2			
		$a_2$	$b_2$	$c_2$	$d_2$
Player 1	$a_1$	6, 2	2,7	1, 4	0,3
	$b_1$	1, 0	3,2	2, 1	1, 1
	$c_1$	7,0	2, 2	1, 5	6, 1
	$d_1$	8,4	1, 2	0, 2	3,9

- (a) Iteratively eliminate strictly dominated strategies for as many steps as possible. In each step, specify which strategy of which player was eliminated and by which strategy it was strictly dominated.
- (b) Specify the set of Nash equilibria in this game. Which action should player 1 play accordingly?

Exercise 2.2 (Minimax strategy profiles, 1.5+1.5 points)

Let G be a two-player zero-sum game that has a Nash equilibrium.

- (a) Show that if some of player 1's payoffs are increased in such a way that the resulting game G' is also a zero-sum game then G' has no Nash equilibrium in which player 1 gets a lower payoff than he got in the Nash equilibria of G.
- (b) Show that the game G' that results from G by elimination of one of player 1's strategies does not have a Nash equilibrium in which player 1's payoff is higher than it is in the Nash equilibria of G.

Exercise 2.3 (Nash equilibria in zero-sum games, 1 point)

Prove the following claim or give a counterexample: If G is a two-player zerosum game that has a Nash equilibrium with payoff v for player 1 then every strategy profile in G with payoff v for player 1 is a Nash equilibrium.

The exercise sheets may and should be worked on and handed in in groups of three students. Please indicate both names on your solution.