Exercise 9.1 (Matching Markets, 3+3)

(a) Consider the market with buyers $B = \{1, 2, 3\}$, sellers $S = \{a, b, c\}$, and valuations

\[
\begin{array}{c|ccc}
  i & a & b & c \\
  \hline
  1 & 6 & 5 & 4 \\
  2 & 5 & 2 & 3 \\
  3 & 4 & 3 & 2 \\
\end{array}
\]

Construct a set of market-clearing prices and find a perfect matching in the resulting preferred-seller graph, using the algorithms from the lecture. Your submission should include all intermediate steps.

(b) Prove the theorem from slide 51 of lecture 14 (Market-Clearing Prices – Optimality II).

Exercise 9.2 (Auctions as Matching Markets, 3+3+3)

We want to model single-item auctions as matching markets. This means there is only one good but $n$ potential buyers. For each potential buyer $b$, let the true valuation for the good be given as $\nu_b$.

(a) Model this auction as a matching market by providing $S$, $B$, and $\nu$.

(b) Assume that there are four agents with true valuations of 3, 3, 2, and 1. From which subset of agents should the winner be chosen and how much should he have to pay? Answer these questions by providing a set of market-clearing prices and identifying a perfect matching in the preferred-seller graph.

(c) How does this result generalize? Provide market-clearing prices for the general case with arbitrary many agents and arbitrary non-negative valuations (but still only one single item) and show that they induce a perfect matching in the PSG.