## Multi-Agent Systems

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## Exercise Sheet 9

## Due: July 17th, 2017, 10:00

Exercise 9.1 (Matching Markets, 3+3)
(a) Consider the market with buyers $\mathcal{B}=\{1,2,3\}$, sellers $\mathcal{S}=\{a, b, c\}$, and valuations

$$
v_{i j}=\begin{array}{c|ccc} 
& \mathrm{a} & \mathrm{~b} & \mathrm{c} \\
\hline 1 & 6 & 5 & 4 \\
2 & 5 & 2 & 3 \\
3 & 4 & 3 & 2
\end{array}
$$

Construct a set of market-clearing prices and find a perfect matching in the resulting preferred-seller graph, using the algorithms from the lecture. Your submission should include all intermediate steps.
(b) Prove the theorem from slide 51 of lecture 14 (Market-Clearing Prices - Optimality II).

Exercise 9.2 (Auctions as Matching Markets, $3+3+3$ )
We want to model single-item auctions as matching markets. This means there is only one good but $n$ potential buyers. For each potential buyer $b$, let the true valuation for the good be given as $\nu_{b}$.
(a) Model this auction as a matching market by providing $\mathcal{S}, \mathcal{B}$, and $v$.
(b) Assume that there are four agents with true valuations of $3,3,2$, and 1. From which subset of agents should the winner be chosen and how much should he have to pay? Answer these questions by providing a set of market-clearing prices and identifying a perfect matching in the preferred-seller graph.
(c) How does this result generalize? Provide market-clearing prices for the general case with arbitrary many agents and arbitrary non-negative valuations (but still only one single item) and show that they induce a perfect matching in the PSG.

