Introduction to Game Theory

B. Nebel, R. Mattmüller
T. Schulte, G. Mouratidis
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University of Freiburg
Department of Computer Science

Exercise Sheet 10
Due: Monday, July 10, 2017

Exercise 10.1 (Top trading cycle method, 2+2+2 points)

(a) Prove that the top trading cycle algorithm returns an allocation that is in the core.

(b) Prove that the top trading cycle mechanism cannot be manipulated.

(c) Apply the top trading cycle algorithm to the following problem and state what happens in the iterations:

- *Player 1:* $1 \lhd_1 4 \lhd_1 2 \lhd_1 3$
- *Player 2:* $3 \lhd_2 2 \lhd_2 1 \lhd_2 4$
- *Player 3:* $2 \lhd_3 3 \lhd_3 4 \lhd_3 1$
- *Player 4:* $2 \lhd_4 1 \lhd_4 4 \lhd_4 3$

Preferences are given from lowest (left) to highest (right).

Exercise 10.2 (Stable matchings, 2 points)

Apply the deferred acceptance algorithm with male proposals to the following problem and state what happens in the iterations:

- *Man 1:* $w_4 \prec_{m_1} w_3 \prec_{m_1} w_1 \prec_{m_1} w_2$
- *Man 2:* $w_3 \prec_{m_2} w_2 \prec_{m_2} w_1 \prec_{m_2} w_4$
- *Man 3:* $w_4 \prec_{m_3} w_2 \prec_{m_3} w_3 \prec_{m_3} w_1$
- *Man 4:* $w_4 \prec_{m_4} w_1 \prec_{m_4} w_3 \prec_{m_4} w_2$
- *Woman 1:* $m_4 \prec_{w_1} m_2 \prec_{w_1} m_3 \prec_{w_1} m_1$
- *Woman 2:* $m_2 \prec_{w_2} m_1 \prec_{w_2} m_4 \prec_{w_2} m_3$
- *Woman 3:* $m_1 \prec_{w_3} m_3 \prec_{w_3} m_2 \prec_{w_3} m_4$
- *Woman 4:* $m_4 \prec_{w_4} m_1 \prec_{w_4} m_2 \prec_{w_4} m_3$

Preferences are given from lowest (left) to highest (right).

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.