Introduction to Game Theory

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Summer semester 2017

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Exercise Sheet 7

Due: Monday, June 19, 2017

Exercise 7.1 (Repeated Games, 4 points)

Consider the infinitely repeated prisoner's dilemma with discount factor $0 < \delta < 1$. The payoff matrix of the stage game is given below.

		Player 2 C	Player 2 D
Player 1	C	$5, 5$	$0, 6$
	D	$6, 0$	$1, 1$

(a) Let t be the *tit-for-tat* strategy as defined in the lecture. Specify the unique run $O(t, t)$ that results from playing t against t.

(b) Compute the discounted payoff $v_1(O(t, t))$ of player 1 for the strategy profile (t, t) for general $0 < \delta < 1$ and for $\delta = \frac{1}{2}$ in particular.

(c) Assume that player 1 deviates from t and plays some other strategy s instead, whereas player 2 still plays t. Without loss of generality, assume further that the first deviation occurs in the first round, where player 1 defects (D) instead of cooperating (C).

Show that, for $\delta = \frac{1}{2}$, we have $v_1(O(s, t)) \leq v_1(O(t, t))$.

(d) Does the result of part (c) give us any Nash equilibria in the infinitely repeated prisoner's dilemma with discount factor $\delta = \frac{1}{2}$?

Exercise 7.2 (Voting procedures, 4 points)

For the following preference relations, determine the winners according to the **plurality vote**, **instant runoff voting**, **Borda count**, and **Coombs method**[1] (for simplicity, we assume that ties are broken in favor of the candidate with the lower index):

2 voters have the preference:	$a_2 \succ a_4 \succ a_3 \succ a_5 \succ a_1$
3 voters have the preference:	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$
1 voter has the preference:	$a_4 \succ a_2 \succ a_5 \succ a_1 \succ a_3$
2 voters have the preference:	$a_5 \succ a_3 \succ a_4 \succ a_2 \succ a_1$

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.

[1] https://en.wikipedia.org/wiki/Coombs%27_method