Introduction to Game Theory

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Exercise Sheet 10

Due: Thursday, July 14, 2016

Exercise 10.1 (House allocation problem, 2+2+2 points)

(a) Apply the top trading cycle algorithm to the following problem and state what happens in the iterations:

- *Player 1:* $1 \lhd_1 4 \lhd_1 2 \lhd_1 3$
- *Player 2:* $3 \lhd_2 2 \lhd_2 1 \lhd_2 4$
- *Player 3:* $2 \lhd_3 3 \lhd_3 4 \lhd_3 1$
- *Player 4:* $2 \lhd_4 1 \lhd_4 4 \lhd_4 3$

Preferences are given from lowest (left) to highest (right).

(b) Prove that the top trading cycle algorithm returns an allocation that is in the core.

(c) Prove that the top trading cycle mechanism cannot be manipulated.

Exercise 10.2 (Stable matchings, 2 points)

Apply the deferred acceptance algorithm with male proposals to the following problem and state what happens in the iterations:

- *Man 1:* $w_4 \prec_{m_1} w_3 \prec_{m_1} w_1 \prec_{m_1} w_2$
- *Man 2:* $w_3 \prec_{m_2} w_2 \prec_{m_2} w_1 \prec_{m_2} w_4$
- *Man 3:* $w_4 \prec_{m_3} w_2 \prec_{m_3} w_3 \prec_{m_3} w_1$
- *Man 4:* $w_4 \prec_{m_4} w_1 \prec_{m_4} w_3 \prec_{m_4} w_2$
- *Woman 1:* $m_4 \prec_{w_1} m_2 \prec_{w_1} m_3 \prec_{w_1} m_1$
- *Woman 2:* $m_2 \prec_{w_2} m_1 \prec_{w_2} m_4 \prec_{w_2} m_3$
- *Woman 3:* $m_1 \prec_{w_3} m_3 \prec_{w_3} m_2 \prec_{w_3} m_4$
- *Woman 4:* $m_4 \prec_{w_4} m_1 \prec_{w_4} m_2 \prec_{w_4} m_3$

Preferences are given from lowest (left) to highest (right).

The exercise sheets may and should be worked on and handed in in groups of two to three students. Please indicate all names on your solution.