Introduction to Game Theory

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Exercise Sheet 8

Due: Thursday, June 30, 2016

Exercise 8.1 (May's theorem, 4 points)

Recall May's theorem: A social choice function $f : L^n \to A$ for a set of two alternatives $A = \{x, y\}$ satisfies anonymity, neutrality and monotonicity iff it is the plurality method (i.e., $f(\prec_1, \ldots, \prec_n) = x$ iff $\#\{i \mid y \prec_i x\} \geq \frac{n}{2}$).

We assume n is odd to avoid tie-breaking issues that could violate neutrality.

Show that each of the three conditions is necessary for May's theorem.

(a) anonymity, i.e., $f(\prec_1, \ldots, \prec_n) = f(\prec_{\pi(1)}, \ldots, \prec_{\pi(n)})$ for all permutations π of the voters $\{1, \ldots, n\}$.

(b) neutrality, i.e., $f(\prec_1, \ldots, \prec_n) = x$ iff $f(\prec'_1, \ldots, \prec'_n) = y$, where $x \prec'_i y$ iff $y \prec_i x$ for all $i = 1, \ldots, n$.

(c) monotonicity, i.e., if $f(\prec_1, \ldots, \prec_n) = x$, then also $f(\prec'_1, \ldots, \prec'_n) = x$, where $\prec'_i = \prec_i$ for $i \neq I$ for some voter I such that $x \prec_I y$ and $y \prec'_I x$.

Hint: For each condition, find a counterexample (a social choice function) that fulfills all other conditions but the one in question and that is not the plurality method.

Exercise 8.2 (Single peaked preferences, 2 + 2 points)

(a) Joe, Max, and Ada discuss how much time to invest in collective preparations for their upcoming exam in game theory. Their valuations over the amount of time $x \in \mathbb{R}^{>0}$ (in hours) to invest are as follows:

$$v_{Joe}(x) = -\frac{7}{3} + \frac{7}{3}x - \frac{1}{15}x^2$$
$$v_{Max}(x) = -\frac{1}{2}x + 20$$
$$v_{Ada}(x) = 4x - \frac{1}{5}x^2$$

To agree on a fixed amount of time $x \in [5, 30]$, Joe, Max, and Ada take a vote in which each of them submits a single peaked preference relation. On what amount of time will they agree using the median rule?

(b) Show that the median rule is not incentive compatible when the preference relations are not restricted to be single peaked. Construct valuation functions for Joe, Max, and Ada, such that at least one of them has an incentive to misrepresent their true preferences.

The exercise sheets may and should be worked on and handed in in groups of two to three students. Please indicate all names on your solution.