

## Introduction to Game Theory

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### Exercise Sheet 7

**Due: Thursday, June 23, 2016**

**Exercise 7.1** (Properties of voting procedures, 4 points)

Consider the voting procedures **plurality vote**, **instant runoff voting**, and the **Borda count**. Again, we assume that ties are broken in favor of the candidate with the lower index. Moreover,  $|A| \geq 3$ . Consider the following properties:

**Majority criterion:** If for more than half of the voters  $i$ ,  $b \prec_i a$  for all  $b \in A \setminus \{a\}$ , then  $f(\prec_1, \dots, \prec_n) = a$ .

**Reversal symmetry:** Let  $\prec_1, \dots, \prec_n \in L$  and for all  $i = 1, \dots, n$ , let  $\prec'_i \in L$  be defined such that for all  $a, b \in A$ ,  $a \prec_i b$  iff  $b \prec'_i a$ . If  $f(\prec_1, \dots, \prec_n) = c$ , then  $f(\prec'_1, \dots, \prec'_n) \neq c$ .

**Incentive compatibility:**  $f(\prec_1, \dots, \prec'_i, \dots, \prec_n) \preceq_i f(\prec_1, \dots, \prec_i, \dots, \prec_n)$  for all  $\prec_1, \dots, \prec_n, \prec'_i \in L$ .

For each of the nine combinations of voting procedure  $f$  and property  $P$ , show that  $f$  satisfies  $P$  or give a counterexample.

**Exercise 7.2** (Schulze method, 4 points)

For the following preference relations determine the set of possible winners according to the Schulze-method<sup>1</sup>:

- 20 voters have the preference  $b \prec_i c \prec_i e \prec_i d \prec_i a$
- 10 voters have the preference  $d \prec_i e \prec_i c \prec_i b \prec_i a$
- 15 voters have the preference  $b \prec_i d \prec_i a \prec_i e \prec_i c$
- 12 voters have the preference  $a \prec_i b \prec_i c \prec_i e \prec_i d$
- 13 voters have the preference  $a \prec_i e \prec_i c \prec_i d \prec_i b$

The exercise sheets may and should be worked on and handed in in groups of two to three students. Please indicate all names on your solution.

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<sup>1</sup>[http://en.wikipedia.org/wiki/Schulze\\_method](http://en.wikipedia.org/wiki/Schulze_method)