Introduction to Game Theory

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Exercise Sheet 3

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Exercise 3.1 (Best response function, 4 points)

Let $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ with $N = \{1, 2\}$, $A_1 = A_2 = \mathbb{R}^{\geq 0}$, $u_1(a_1, a_2) = a_1(a_2 - a_1)$ and $u_2(a_1, a_2) = a_2(1 - \frac{1}{2}a_1 - a_2)$ for all $(a_1, a_2) \in A$.
Define all Nash equilibria of this game by constructing and analyzing the best response function of both players.

Exercise 3.2 (Kakutani's fixed point theorem, 1+1+1+1 points)

Let X be a compact, convex, non-empty subset of $\mathbb{R}^n$ and let $f : X \to 2^X$ be a set-valued function for which

- for each $x \in X$, the set $f(x)$ is nonempty and convex;
- the graph of f is closed (i.e. for all sequences $\{x_k\}$ and $\{y_k\}$ such that $y_k \in f(x_k)$ for all k, $x_k \to x$, and $y_k \to y$, we have $y \in f(x)$).

Then there exists $x^* \in X$ such that $x^* \in f(x^*)$.

Show that each of the following four conditions is necessary for Kakutani's theorem.

(a) X is compact.

(b) X is convex.

(c) $f(x)$ is convex for each $x \in X$.

(d) f has a closed graph.

Hint: There exists a counter-example with $n = 1$ in each of the four cases.

The exercise sheets may and should be worked on and handed in in groups of two to three students. Please indicate all names on your solution.