Exercise Sheet 11
Due: 15-07-2015

The aim of this sheet is to implement a tableaux solver for the modal logic $K$. You may worked, and hand in, your solutions in groups of at most three students! In that case please indicate all names in your solution. This week the solution must be handed in on Wednesday before the lecture by email to wenzelmf@tf.uni-freiburg.de.

You may use one of the programming languages Python 3, Java, C++. On the website of the lecture you will find a working Python 3 implementation of a parser and the internal representation of formulae (see (a) and (d)). You may use this code, but you don’t need to.

Exercise 11.1 (1+1+2+1+5 points)

(a) Implement an internal representation of $\mathcal{L}_K(P)$-formulae of the form

$$\varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid \varphi \rightarrow \varphi \mid \varphi \leftrightarrow \varphi \mid \Box \varphi \mid \Diamond \varphi.$$ 

(b) Implement a procedure that eliminates from a given $\mathcal{L}_K(P)$-formula all occurrences of $\rightarrow$ and $\leftrightarrow$, by replacing formulae of the form $(\varphi \rightarrow \psi)$ by $(\neg \varphi \lor \psi)$ and $(\varphi \leftrightarrow \psi)$ by $((\neg \varphi \lor \psi) \land (\varphi \lor \neg \psi))$, respectively.

(c) Implement a procedure that transforms a given formula into negation normal form (NNF). You may assume that the input formula does not have any occurrences of $\rightarrow$ and $\leftrightarrow$.

(d) Write a parser that reads formulae and transforms them into the internal representation (implemented in (a)). The formulae are input as strings with the following notation:

- $S \rightarrow p_1 \mid p_2 \mid p_3 \mid \ldots$ for propositional variables
- $S \rightarrow \mid S S$ for disjunctions
- $S \rightarrow \& S S$ for conjunctions
- $S \rightarrow \rightarrow S S$ for implications
- $S \rightarrow \leftrightarrow S S$ for equivalences
- $S \rightarrow \neg S$ for negations
- $S \rightarrow \Box S \mid \Diamond S$ for boxes and diamonds.

(e) Implement the tableaux method for deciding the $K$-validity of an input formula.

Input:
Your program should be called from the command line by:

```
# tableau <formula>
```

as in the following example (which checks the validity of axiom $K$):

```
# tableau "\rightarrow \Box \rightarrow p_1 p_2 \rightarrow \Box \neg p_1 \Box p_2"
```

Output:
The last line of the output of your program should be "TRUE", when the input formula is $K$-valid, and "FALSE" otherwise.