can we transfer this (impossibly) result to social choice function?
Yes!
It is even worse:
We can show that it is impossible
for any social choice function to
be immune to 

\[ f: \mathbb{R}^n \rightarrow \mathbb{R} \]

\[ f(c_1, \ldots, c_n) = c_i \]

and 

\[ a \preceq b \]

implies 

\[ f(a_1, \ldots, a_n) \preceq f(b_1, \ldots, b_n) \]

Proposition
A social choice function is 

\[ \text{intransitive} \]

if it is 

\[ \text{intransitive}. \]

Proof:
Let \( f \) be intransitive: it \( f(c_1, \ldots, c_i, \ldots, c_n) = a \) and 

\[ f(c_1, \ldots, c', \ldots, c_n) = b \]

and \( a \preceq c_i \preceq b \preceq a \)
and \( a \preceq b \). Then \( f \) cannot be transitive since \( a \preceq c_i \preceq b \preceq a \) and 

\[ f(c_1, \ldots, c_i, \ldots, c_n) = a \]

and 

\[ f(c_1, \ldots, c', \ldots, c_n) = b \] and \( a < b. \)

\[ \text{Suppose intransitivity is violated, i.e.,} \]

\[ \exists a, b \in A: f(c_1, \ldots, c_i, \ldots, c_n) = a \text{ and} \]

\[ f(c_1, \ldots, c', \ldots, c_n) = b \text{ and} \]

\( a \preceq c_i \preceq b \preceq a \) and 

\( a \preceq b \). If \( a \preceq c_i \preceq b \preceq a \) and 

\[ a \preceq b. \]

then assume \( a \preceq c_i \preceq b \) do the preference \( c_i \) 

\[ \text{in manipulate again}. \]
Def
A function \( f: X \to Y \) is called **dictatorial** on \( X \) if for all \( y \in Y \) there exists \( x \in X \) such that \( f(x) = y \).

Def (Dictatorship)
A function \( f: X \to Y \) is called a **dictator** on \( X \) if for all \( x, y, z \in X \) such that \( f(x) = y \) and \( f(z) = y \), then \( x = z \).

Theorem (Borda - Substitutivity)
If \( f \) is an incentive compatible and onto social function over at least three alternatives, then \( f \) is a dictatorship.

Proof:
- Use Arrow's Theorem.
- Construct a social welfare function from a social choice function.
- Construction is clear (i.e., directly from an incentive compatible and onto and non-dictatorial social choice function).
- The construction ensures that the social welfare function satisfies IIA, monotonicity, and non-dictatorial.