Axioms for Voting Systems

We discuss them for Plurality, IRV, Borda, Schulze.

Neutralty: "Names" of candidates should not be relevant.

Anonymity: "Names" of the votes should not be relevant.

PV, Borda in IRV, Schulze.

Majority criterion: A candidate who is ranked highest by the majority (>50%) of votes, should be the winner.

Borda: —
Plurality, IRV, Schulze: +

Mutual majority criterion: If a majority of votes prefers candidates from one subset S over all other candidates, then the winner should be from S.

Independence of Irrelevant Alternatives (IIA):
The outcome never changes if a non-winning candidate is added or removed.

Borda: —
Plurality: —
IRV: —
Schulze: —
Independence of Clone Attractions

A set of clones is a subset of candidates S such that all other candidates are ranked higher or lower than all candidates from S by all votes.

Schulze, IRV: +
Borda, Plurality: —

Polyomial-time computability: The winner should be computable in time polynomial in the number of candidates and linear in the number of votes.

Borda, Plurality, IRV, Schulze: +
Dodgson: —

Monotonicity criterion: If a candidate wins an election, then he will still win, if one vote raises him higher.

Borda, Plurality, Schulze: +
IRV: —

Consistency criterion: If there are two sets of votes, with separate elections that have the same winner, then the combined election should have the same winner.

Resolvability:

1) For easy tie between "winners", one vote should resolve the tie.

2) The proportion of preference profiles leading to a tie should approach zero when the number of votes approaches infinity.

Plurality, IRV, Borda, Schulze: +
**Theorem (May, 1958)**: A voting method for two alternatives satisfies anonymity, neutrality, and monotonicity, if it is the plurality method.

**Proof:**

"\(\iff\)" : For simplicity, we assume that the number of votes is odd.

\[ \text{Arrow's Impossibility Theorem} \]

For three or more alternatives, there are no voting methods that satisfy a small set of reasonable criteria.

**Arrow's Impossibility Theorem**

**Theorem (Arrow)**: Every social welfare function based on more than two alternatives that satisfies anonymity and IIA is necessarily a dictatorship.
**Def.: (Total comparability)**

For all \( x \in L \): \( F(x, x, \ldots, x) = < \).

**Def.: (Partial comparability)**

For all \( x_1, x_2, \ldots, x_n, x \in L \) with
\( x = F(x_1, \ldots, x_n) \), if \( a \leq b \) for all \( i = 1, \ldots, n \), then also \( a \leq b \).

**Remark:** Partial comparability implies total comparability, but not the other way around.