Is the grim strategy a NE?

\[ V_1(0, g, g) = 3 - \frac{x}{4} \geq 3 \cdot \left( 1 + \frac{x}{4} \right) \]

\[ D(0, g, g) = \left( 0, 0, 0, \ldots \right) \]

\[ \frac{x}{4} + x \leq 3 \cdot \left( 1 + \frac{x}{4} \right) \]

\[ x \leq \frac{24}{1 - \frac{x}{4}} \]

\[ \frac{4 + \frac{x}{4}}{1 - \frac{x}{4}} \leq 3 + \frac{3x}{1 - \frac{x}{4}} \]

If this is good for \( g \), then \( \frac{x}{4} \) is not bounded away from \( g \).

Positive message: In repeated games, there are other NEs than the (0, 0).

Negative message: Which one do you play?
Social Choice Theory

- Aggregation of preferences of group members.
- Voting and voting protocols.
  - Elections.
  - Committee decisions.
  - European Union Council.

**Example:**

The voters: 1, 2, 3
Candidates: a, b, c

1 > 2 > 3
b > a, c > a, c > b
b > c, a < b, c < b, ...

If we have many voters, we can specify how many vote in one list:

<table>
<thead>
<tr>
<th>Vote</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

Voting protocols

1) What do the voters have to do submit (and when)?
2) How do computer the result?

Plurality (the "first-past-the-post," "winner-takes-it-all")
- Voters submit only their top preference.
- Candidate with most votes wins.

Drawbacks: winner may not have more than 50%.
- Plurality with runoff.
- Voters submit 2 preferences.
- The two candidates with most go to second round.
- In the second, voters decide between the two candidates.
In many runoffs (in most votes)
- each voter gives full preference list
- iteratively candidates with the least number
  of top preferences are eliminated until only
  one candidate remains.

- Borda count
  - each voter submits his preference order
  - if a candidate is in position $j$ of a voter's
    list, he gets $n-j$ points from that voter
  - points are added up
  - the one with the most points wins.

- Condorcet winner
  - each voter submits his preference
  - perform pairwise comparison between
    candidates (how many votes for and the
    one over the other).
  - if one candidate wins all the comparisons,
    he is the winner.

- Drawback: This winner might not exist because
  the resulting order is cyclic.

In general, all methods might come up with
draws, but this is usually ignored because in
large elections it is very unlikely.

Running Example:
23 voters, candidates: a, b, c, d, e

\[
\begin{array}{c|cccc}
\text{Vote} & a & b & c & d \\
\hline
1 & 6 & 4 & 3 & 2 \\
2 & 3 & 6 & 4 & 1 \\
3 & 5 & 3 & 6 & 1 \\
4 & 2 & 5 & 1 & 6 \\
5 & 4 & 2 & 1 & 5 \\
\end{array}
\]

\text{Plurality:}\ e \ \text{wins, because}\ e \ \text{has 8 votes (out of 23)}

Running Example:
23 voters, candidates: a, b, c, d, e

\[
\begin{array}{c|cccc}
\text{Vote} & a & b & c & d \\
\hline
1 & 6 & 4 & 3 & 2 \\
2 & 3 & 6 & 4 & 1 \\
3 & 5 & 3 & 6 & 1 \\
4 & 2 & 5 & 1 & 6 \\
5 & 4 & 2 & 1 & 5 \\
\end{array}
\]

\text{Plurality:}\ e \ \text{wins, because}\ e \ \text{has 8 votes (out of 23)}
Running Example:

23 votes, candidates: a, b, c, d, e

<table>
<thead>
<tr>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
</tr>
<tr>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>d</td>
<td>e</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>d</td>
<td>e</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>e</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
</tr>
</tbody>
</table>

5 votes for e, 4 votes for a, 3 votes for b, 2 votes for c, 1 vote for d

5 votes for e in 3rd place
4 votes for a in 2nd place
4 votes for b in 1st place
2 votes for c in 5th place
1 vote for d in 4th place

Final tally:
e: 5 votes
a: 4 votes
b: 4 votes
c: 2 votes
d: 1 vote

1st round: e in lead
2nd round: Eliminate d
3rd round: Eliminate a
Now b has 4 votes

\[ \rightarrow b \text{ wins} \]
Different winners for different voting protocols?

Which one should be used?

Choice of the voting protocol can be used strategically.

Schulze method

Condorcet method

Has a technique for breaking

Usually, Round Panic, others use it