One Deviation Property:

Let $\Gamma = \langle N, A, H, P, (u_i)_{i \in N} \rangle$ be a finite-horizon EGUEP. Then a strategy profile $s^*$ is an SPE of $\Gamma$ if for every player $i \in N$ and every history $h \in H$ with $P(h) = i$, we have

$$u_i(h, (O_i(s_{-i|h}, s_{i|h})) \geq u_i(h, (O_i(s_{-i|h}, s_i)))$$

for every strategy $s_i$ of player $i$ in the subgame $\Gamma(h)$ that differs from $s_{i|h}$ only in the action after the shifted history of $T(h)$. 
In infinite-horizon games, the one-deviation property does not hold.

**Counterexample (2-player game):**

\[ \begin{array}{cccccc}
A & D & \downarrow & D & \cdots & A \\
\uparrow & D & \downarrow & D & \cdots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\end{array} \]

Strategy \( s_n (h) = D \) for all \( h \in H \setminus \mathbb{Z} \).

Now, \( s_n \) is not a SPE.

For all histories \( h' \), there is no profitable deviation from \( s_n | h' \) that gives higher payoff than \( D \).

But strategy \( s^*_n (h) = A \) for \( h \in H \setminus \mathbb{Z} \) is a SPE. (D)
Theorem (Kuhn): Every finite extermne game with perfect information has a SPE.

Proof idea:

* Constructive: build a SPE bottom-up (backward induction)

* Similar to minimax procedure from intro to AI class.
Example:

Diagram with labeled nodes and arrows.
Proof: Let $\Gamma = (N, A, H, P, (u_i))$ be a finite EGUPE. Construct a SPE by induction on $l(\Gamma(h))$ for subgame $\Gamma(h)$. In parallel, construct functions $t_i : H \rightarrow \mathbb{R}$ for all players $i \in N$ s.t. $t_i(h)$ is the payoff of player $i$ in a SPE in subgame $\Gamma(h)$. Base case: If $l(\Gamma(h)) = 0$, then $t_i(h) = u_i(h)$ for all $i \in N$. Inductive case: Assume that $t_i(h)$ is already defined for all $h \in H$ with $l(\Gamma(h)) \leq k$. Consider
history $h^* \in H$ with $\ell(\Gamma(h^*)) = k+1$.
Assume $\rho(h^*) = i$. Let

$$s_i(h^*) := \arg\max_a t_i(h^*, a),$$
$$a \in A(h^*)$$

$$t_j(h^*) := t_j(h^*, s_i(h^*))$$

for all $j \in N$.

Inductively, we obtain a strategy profile that satisfies the one-deviation property (by construction). From the 1-dev-prop. lemma, it follows that the constructed profile is indeed a SPE. \(\Box\)