One Deviation Property:
Let $\Gamma = (N, A, H, P, (\mathcal{U}_i)_{i \in N})$ be a finite-horizon EAGRE. Then a strategy profile $s^*$ is an SPE of $\Gamma$ if for every player $i \in N$ and every history $h \in H$ with $P(h) = i$, we have

$$u_i(h, (s^*_{-i}h, s^*_ih)) \geq u_i(h, (s^{\pi}_ih, s^*_ih))$$

for every strategy $s^\pi_i$ of player $i$ in the subgame $\Gamma(h)$ that differs from $s^*_ih$ only in the action after the initial history of $\Gamma(h)$.

For infinite-horizon games, the one-deviation property does not hold.

\textbf{Counterexample (2-player game):}

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Consider strategy $s_i(h) = A$ for all $h \in H \setminus 1$.

Now, $s_i$ is not an SPE.

For all histories in $H$, there is no profitable deviation from $s_i$. But deviating from $s_i$ gives higher payoff than $D$.

\textbf{Theorem (Kuhn):} Every finite extensive game with perfect information has a SPE.

\textbf{Proof Idea:}

- Constructive: build a SPE bottom-up (backward induction)
- Similar to minimax procedure from info to AE class.

\textbf{Example:}
Proof: Let \( \Gamma = (N, A, H, P, (a_i)) \) be a finite ENEPE. Construct a SPE by induction on \( l(\Gamma(h)) \) for \( h \in H \). Let \( \Gamma(h) \) be the strategy profile for all players \( i \in N \) s.t. \( t_i(h) \) is the payoff of player \( i \) in a SPE in subgame \( \Gamma(h) \). Base case: If \( l(\Gamma(h)) = 0 \), then \( t_i(h) = u_i(h) \) for all \( i \in N \).

Inductive case: Assume that \( t_i(h) \) is already defined for all \( h \in H \) with \( l(\Gamma(h)) \leq k \). Consider a history \( h^* \in H \) with \( l(\Gamma(h^*)) = k+1 \). Assume \( P(h^*) = i \). Let

\[
\begin{align*}
  s_i(h^*) &= \arg\max_{a \in A_i(h^*)} t_i(h^*, a) \\
  t_j(h^*) &= \max_{a \in A_j(h^*, s_i(h^*))} t_j(h^*, a)
\end{align*}
\]

for all \( j \in N \).

Inductively, we obtain a strategy profile that satisfies the one-deviation property (by construction). From the induction step, it follows that the constructed profile is indeed a SPE. \( \square \)