Extensive Games

Finite game means the set H is finite.

Finite horizon was defined as "no infinite history".

Osborne/Rubinstein: "If the longest history is finite, all histories are finite."

... infinite branch.

This is not finite horizon.

The "right" definition: "If there exists an upper bound for the length of the histories, then the game is a finite horizon game."
Strategies in extensive games

Strategies = actions

Def (strategy)

Let $\Gamma = \langle N, A, H, P, \{(u_i)\} \rangle$ be a $EVWPNI$. Then the set of actions $a$ with $(h, a) \in \Gamma$ are denoted by $A(h)$. A strategy of player $i$ is a function $s_i$ that assigns to each non-terminal history $h \in H \setminus Z$ with $P(h) = i$ an action $a \in A(h)$. The set of strategies of player $i$ is denoted by $S_i$.

Remark: Strategies require us to assign actions to histories, even if it is clear they will never be played.
Notation: Strategies are often given by writing the actions going through the game tree in a level-by-level, left-to-right way:

Example:

![Game Tree Diagram]

Strategies for player 1
AEE, AF, BE, BF

Strategies for player 2
C, D

Def (outcome)

The outcome of a strategy profile \( S = (S_i)_{i \in N} \) is the history \( h^S = (a_k)_{k=N}^L \) such that for all \( 0 \leq k \leq L \), \( L \in \mathbb{N} \cup \{\infty\} \), where \( S(I(a_1, \ldots, a_k)) (a_1, \ldots, a_k) = a_{k+1} \).
The outcome of strategy $s$ is denoted by $O(s)$.

**Example**

$$O((A, F, D)) = (A, D)$$
$$O((A, F, C)) = (A, C, F)$$

**Def (NIE)**

A Nash Equilibrium of an extensive game with perfect information $Γ$ is a strategy profile $s^* = (s^*_i)_{i ∈ N}$ such that for each player $i ∈ N$:

$$u_i(O(s^*)) ≥ u_i(O(s^*_{-i}, s_i))$$

for all $s_i ∈ S_i$. 
\textbf{Def} (game induced by extensive game)

The \textit{stochastic} game \( G' \) induced by an \textit{extensive} game \( \Gamma \) is defined by

\[ G' = \langle N', (A_i'), \pi, (u_i') \rangle \quad \text{with} \]

\[ A_i' = S_i, \quad A' = S \]

\[ u_i'(a) = u_i'(O(s)) \]

\textbf{Proposition}

The NE of an EGW \( \Gamma \) are exactly the NE of the induced stochastic game \( G' \).
Remarks:

1) Each E6 WPI can be transformed into a strategic game, but the created game may be exponentially larger.

2) The other direction does not work (because we do not have simultaneous actions).
Noncredible threat

NE, but a very funny one.