Extensive Games

Finite game means the set \( H \) is finite. 0

Finite horizon was defined as "no infinite history" 0

Osborne/Rafelski: "If the largest history is finite, then all stories are finite, too."

The right definition: "If there exists an upper bound for the length of the histories, then the game is a finite horizon game."

Strategies in extensive games

Strategies & action s

Def (strategy)

Let \( \Gamma = \langle N, A, H, P, u \rangle \) be a EGP. Then the set of actions a with \( (h,a) \in H \) are denoted by \( A(h) \). A strategy of player \( i \) is a function \( s_i : H \rightarrow A(i) \) that assigns to each non-terminal history \( h \in H \setminus C \) with \( P(h) \) an action as \( A(i) \). The set of strategies of player \( i \) is denoted by \( S_i \).

Remark: Strategies require no decision and no delay histories, even if it is clear they will never be played!

The outcome of strategy \( s \) is denoted by \( O(s) \).

Example

Def (NE)

A Nash Equilibrium of an extensive game with perfect information \( \Gamma \) is a strategy profile \( s^* = (s_i^*) \) such that for each player \( i \in N \):

\[ u_i(O(s^*)) \geq u_i(O(s^*_{-i}, s_i)) \]

for all \( s_i \in S_i \).
**Definition:**

The strategic game \( G' \) induced by an extensive game \( G \) is defined by:

\[ G' = (N', \{A'_i\}, \{v'_i\}) \]

where

\[ A'_i = S, \quad A' = S \]

\[ v'_i(a) = v_i (O(a)) \]

**Proposition:**

The NE of an EGWRI \( G \) are exactly the NE of the induced strategic game \( G' \).

**Remarks:**

1. Each EGWRI can be transformed into a strategic game, but this created game may be exponentially larger.
2. The other direction does not work because we need kn Online Simultaneous Actions.