12. Resource Allocation

12.1 Motivation

What we’ve learned so far

Last time we learned:
- Coalition Games with Goals
  - Goals, not numeric utilities, as targets for agents
  - Qualitative coalition games
  - Coalition resource game
- Coalition Structure Formation
  - Maximizing social welfare, instead of individual agent’s utility
  - Number of coalition structures exponential in the number of coalitions

Today: Resource Allocation
Resource allocation: background

The situation:

▶ Only scarce resources available
▶ More than one agent interested in resources

⇒ How to allocate resources efficiently, i.e. allocate them to those agents that value them the most?

Auctions are a solution; different types introduced today:

▶ English auctions
▶ Dutch auctions
▶ First-price sealed-bid auctions
▶ Vickrey auctions
▶ Combinatorial auctions

Classifying auctions

Auction protocol and strategy are effected by several factors:

1. Value of good:
   ▶ public/common (standard one dollar bill)
   ▶ private (bill signed by Bill Clinton), or
   ▶ correlated (special bill, but reselling value also important)

2. Auction protocol:
   ▶ Winner determination: first-price or second-price auction
   ▶ Bidding procedure: open cry or sealed-bid
   ▶ Mechanism: one-shot or ascending/descending

3. Single versus multiple items

Next, private/correlated, first-price, open-cry, ascending, single item auction:

⇒ English auction

12.2 Single Item Auctions

English auctions

<table>
<thead>
<tr>
<th>Auction</th>
<th>Action protocol</th>
<th># items</th>
</tr>
</thead>
<tbody>
<tr>
<td>English auction</td>
<td>first-price, open cry, one-shot, ascending</td>
<td>Single</td>
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</table>

English auction (EA) perhaps the most commonly known type of auction (Sotheby's):

Procedure:

1. Auctioneer suggests reservation price (may be zero)
2. Agents must bid more than the current highest bid
3. All agents see the bids being made and can place bids at any time
4. No more bids ⇒ current highest bid wins and agent has to pay amount of his bid

If value is correlated, counterspeculation can occur

Dominant strategy in private EA: bid a small amount above highest current bid until one's own valuation reached

Winner's curse: Why did no other agent value the good so highly? Did I pay too much?
### Dutch auctions

<table>
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<tbody>
<tr>
<td>Dutch auction</td>
<td>first-price, open cry, one-shot, descending</td>
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Dutch auction (DA):

- Procedure:
  1. Auctioneer starts with *artificially high value* much above the expected value of any bidder’s valuation.
  2. Auctioneer continuously lowers the offer price by small value until...
  3. Some agent makes a bid for the good equal to the current offer price.
  4. The agent has to pay amount of his bid.

- DA is also susceptible to *winner’s curse*.

### First-price, sealed-bid auctions

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</tr>
</thead>
<tbody>
<tr>
<td>First-price sealed-bid</td>
<td>first-price, sealed-bid, one-shot</td>
<td>single</td>
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First-price sealed-bid auction is simplest of all auctions considered here:

- Procedure:
  1. Single round, in which bidders submit their bids privately to the auctioneer.
  2. Auctioneer awards good to agent with *highest bid*.
  3. The agent has to pay amount of his bid.

- Dominant strategy: Bid less than its true value.
- Problem: How much less?
- No general solution as it depends on the other agents.

### Vickrey auctions

<table>
<thead>
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<th># Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vickrey auction</td>
<td>second-price, sealed-bid</td>
<td>Single</td>
</tr>
</tbody>
</table>

Vickrey auctions:

- Probably the most counterintuitive auction type.
- Procedure:
  1. Single round, in which bidders submit their bids privately to the auctioneer.
  2. Auctioneer awards good to agent with *highest bid*.
  3. The agent has to pay amount of *second-highest bid*!

- Dominant strategy: Bidders bid their true valuations.
- Not prone to strategic manipulation.
- Not very popular in real life, but very successful in computational auction systems.
- Problem: anti-social behavior might occur.

### Expected revenue

The expected revenue of the auctioneer depends on attitudes of auctioneers and bidders:

- **Risk-neutral bidders:** revenue provably identical in all four auctions (under certain simple assumptions).
- **Risk-averse bidders:** Dutch and first-price sealed-bid auctions best for auctioneer’s revenue as risk-averse bidders ‘insure’ themselves by bidding slightly more than true valuation.
- **Risk-averse auctioneers:** Prefer Vickrey or English auction over first-price sealed-bid and Dutch.

Important:

- For first result *private values* must exist in agents.
- In general, auction scenario must carefully be analyzed when choosing auction protocol.
Lies and collusion

Ideally:
1. **auctioneer** wants a protocol to be immune to collusions by bidders
2. **bidders** want **honesty** to be dominant strategy for auctioneer

Solutions:
1. immune to collusions ⇒ bidders don’t know each other
2. honest auctioneer ⇒ open-cry auctions or third party handles bids (esp. in case of second price auction)

Further opportunity for auctioneer to manipulate: place bogus bidders, known as *shells* to realize *shill bidding*

⇒ esp. problematic in online auctions such as ebay

### Single Item Auctions overview

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<tr>
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<th>Auctioneer’s revenue best when</th>
</tr>
</thead>
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<tr>
<td>English auction</td>
<td>first-price, open-cry, one-shot, ascending</td>
<td>auctioneer’s risk-averse</td>
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Counter speculation:
- bidders try to gain information either about *true value* of good, or about the *valuations* of other bidders
- If *free* and *accurate*, then every agent would do it
- Otherwise, only if agent’s expected result with costly counterspeculation *no worse* than result without

### 12.3 Combinatorial Auctions

- Bidding languages
- Winner determination
- VCG mechanism

Combinatorial Auctions

Vickrey auctions work well for single items. How about **resources that are divisible**?

⇒ Combinatorial auctions:
- Generalized model of resource allocation
- Auctioning **bundles of goods** $Z = \{z_1, \ldots, z_n\}$ (e.g. frequency bands of the mobile phone network)
- New *valuation function* $v_i : 2^Z \to \mathbb{R}$ indicates how much each $Z \subseteq Z$ is worth to agent $i$
- Important properties of valuation functions:
  - **Normalization**: $v(\emptyset) = 0$
  - **Free disposal**: $Z_1 \subseteq Z_2 \Rightarrow v(Z_1) \leq v(Z_2)$
- **Outcome**: allocation $Z_1, Z_2, \ldots, Z_n$ of goods being auctioned among the agents
Combinatorial Auctions & social welfare

One natural property combinatorial auctions should satisfy is

⇒ maximization of social welfare

\[
V^* = \arg \max_{V} \left( \sum_{i=1}^{n} \alpha_i - \beta_i \right)
\]

where \( \alpha_i \) and \( \beta_i \) are the values and costs of the items.

⇒ Winner determination: computing the optimal allocation \( V^* \) given the valuations submitted by bidders

⇒ Strategic manipulation: agents may not reveal their true valuations (e.g., may overstate the value of bundles)

⇒ Representational complexity: exponential in the number of goods (listing all possible valuations of all bundles)

⇒ Computational complexity: winner determination is NP-hard even under restrictive assumptions

Combinatorial Auctions

Bidding languages

As before, most succinct representation schemes for valuation function preferred; first option: Atomic bid

⇒ \( \beta = (Z, p) \), where \( Z \subseteq \mathbb{Z} \) and \( p \in \mathbb{R}_+ \) is the price

⇒ A bundle of goods \( Z' \) satisfies \((Z, p)\) if \( Z \subseteq Z'\), e.g.:

- Bundle \( \{a, b, c\} \) satisfies the atomic bit \((\{a, b\}, 4)\)
- Bundle \( \{b, d\} \) does not satisfy the atomic bit \((\{a, b\}, 4)\)

⇒ An atomic bid \( \beta = (Z, p) \) defines the valuation function \( v_\beta \):

\[
v_\beta(Z') = \begin{cases} 
  p & \text{if } Z' \text{ satisfies } (Z, p) \\
  0 & \text{otherwise}
\end{cases}
\]

⇒ Not sufficient to express very interesting valuation functions

OR bids

OR bids: Combine more than one atomic statement disjunctively

⇒ \( \beta = (Z_1, p_1) \lor ... \lor (Z_k, p_k) \), for example:

\[
\beta_1 = (\{a, b\}, 3) \lor (\{c, d\}, 5)
\]

⇒ "I would pay 3 for a bundle that contains \( a \) and \( b \) but not \( c \) and \( d \); 5 for a bundle with \( c \) and \( d \) but not \( a \) and \( b \); and 5 for a bundle with \( a, b, c \) and \( d \)."

⇒ Formally:

\[
v_\beta(Z') = \begin{cases} 
  0 & \text{if } Z' \text{ does not satisfy any of } (Z_1, p_1), \ldots, (Z_k, p_k) \\
  \max(p | Z_i \subseteq Z') & \text{otherwise}
\end{cases}
\]

⇒ XOR bids are fully expressive

⇒ number of bids may be exponential in \(|Z|\)

⇒ \( v_\beta(Z) \) can be computed in polynomial time

XOR bids

XOR bids: Specify a number of bids, but par for at most one

⇒ \( \beta_1 = (\{a, b\}, 3) \lor (\{c, d\}, 5) \) ⇒ "I would pay 3 for a bundle that contains \( a \) and \( b \) but not \( c \) and \( d \); 5 for a bundle with \( c \) and \( d \) but not \( a \) and \( b \); and 5 for a bundle with \( a, b, c \) and \( d \)."

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Bidding languages

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Winner determination I

Winner determination is combinatorial optimization problem ⇒ find sets of goods that maximizes some valuation function:

- Proven to be NP-hard in worst case
- Optimal solution calculated using standard technique ⇒ integer linear programming:
  - objective function to maximize: \( f(x_1, \ldots, x_n) \)
  - subject to constraints:
    \[ \phi_1(x_1, \ldots, x_k), \phi_2(x_1, \ldots, x_k), \ldots, \phi_l(x_1, \ldots, x_k) \]
- With set \( Z \) of goods, set \( Ag = \{1, \ldots, n\} \) of agents, and valuation functions \( v_1, \ldots, v_n \) (one per agent), \( Z \subseteq \mathbb{Z} \):
  - introduce variables \( x_i, Z \), with \( x_i, Z = 1 \), if bundle \( Z \) is allocated to agent \( i \), otherwise \( x_i, Z = 0 \)
  - Note: many such variables need to be introduced!

The VCG mechanism

Naïve mechanisms are prone to strategic manipulation, thus ⇒ design mechanism such that, if agents act rationally, dominant strategy is (again) to tell true valuation function

Vickrey-Clarke-Grooves mechanism (VCG mechanism) is generalization of Vickrey's auction from single to divisible goods

Terminology:
- 'Indifferent' valuation function \( v^0(Z) = 0 \) for all \( Z \subseteq \mathbb{Z} \)
- social welfare of all agents but \( i \)

VCG mechanism II

The Vickery-Clarke-Grooves mechanism:
1. Agents declare valuation functions \( \hat{v}_i \) (may not be true)
2. Mechanism chooses allocation maximizing social welfare:
   \[ Z_1^*, \ldots, Z_n^* = \arg \max_{(Z_1, \ldots, Z_n) \in \text{alloc}(\mathbb{Z} Ag)} sv(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, \hat{v}_n) \]
3. Every agent pays to the mechanism or receives from it an amount \( p_i \):
   - compensation' for the utility other agents lose by \( i \) participating, or
   - 'reward' for improving the overall utility (then \( p_i < 0 \))

\[ p_i = sv_{\hat{v}}(Z_1^*, \ldots, Z_n^*, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n) - sv_{\hat{v}}(Z_1^*, \ldots, Z_n^*, \hat{v}_1, \ldots, \hat{v}_n), \text{ where} \]
\[ Z_1^*, \ldots, Z_n^* = \arg \max_{(Z_1, \ldots, Z_n) \in \text{alloc}(\mathbb{Z} Ag)} sv(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n) \]
VCG mechanism III

Properties of the VCG mechanism:
- VCG mechanism is incentive compatible, i.e. telling the truth is dominant strategy
- For a single goos VCG mechanism reduces to Vickrey mechanism
  \[ \Rightarrow p_i \text{ would be the amount of second highest valuation} \]
- Computing VCG payments \( p_i \) is NP-hard

VCG mechanism shows that
\[ \Rightarrow \text{social welfare maximization can be implemented in dominant strategies in combinatorial auctions!} \]

Summary

What we have learned today:
- Different auction types, protocols, and properties thereof
  - English, Dutch, First-price sealed-bid, and Vickrey auction
  - open cry versus sealed-bid, ascending versus descending
  - honesty & collusion
- Combinatorial auctions
  - valuation functions & their properties
  - maximization of social welfare
  - Bidding languages
  - Winner determination
  - The VCG mechanism

Next: Bargaining

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  http://www.inf.ed.ac.uk/teaching/courses/abs/abs-timetable.html