Multiagent Systems
12. Resource Allocation

B. Nebel, C. Becker-Asano, S. Wölfl

Albert-Ludwigs-Universität Freiburg

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12.1 Motivation

What we've learned so far

Last time we learned:
- Coalition Games with Goals
  - Goals, not numeric utilities, as targets for agents
  - Qualitative coalition games
  - Coalition resource game
- Coalition Structure Formation
  - Maximizing social welfare, instead of individual agent's utility
  - Number of coalition structures exponential in the number of coalitions

Today: Resource Allocation

Resource allocation: background

The situation:

- Only **scarce resources** available
- **More than one agent** interested in resources

⇒ How to allocate resources **efficiently**, i.e. allocate them to those agents that value them the most?

Auctions are a solution; **different types** introduced today:

- English auctions
- Dutch auctions
- First-price sealed-bid auctions
- Vickrey auctions
- Combinatorial auctions

Classifying auctions

Auction protocol and strategy are effected by several factors:

1. Value of good:
   - **public/common** (standard one dollar bill)
   - **private** (bill signed by Bill Clinton), or
   - **correlated** (special bill, but reselling value also important)
2. Auction protocol:
   - Winner determination: **first-price** or **second-price** auction
   - Bidding procedure: **open cry** or **sealed-bid**
   - Mechanism: **one-shot** or **ascending/descending**
3. **Single** versus **multiple** items

Next, **private/correlated**, **first-price**, **open-cry**, **ascending**, **single item** auction:
⇒ English auction

12.2 Single Item Auctions

English auctions

Auction	Action protocol	# items
English auction	first-price, open cry, one-shot, **ascending**	single

English auction (EA) perhaps the most commonly known type of auction (Sotheby's):

- Procedure:
  1. Auctioneer suggests **reservation price** (may be zero)
  2. Agents must bid **more than the current highest bid**
  3. All agents **see the bids** being made and can place **bids at any time**
  4. No more bids ⇒ current **highest bid wins** and agent has to pay amount of his bid
- If value is correlated, **counterspeculation** can occur
- **Dominant strategy** in private EA: bid a small amount above highest current bid until one's own valuation reached

Winner's curse: Why did no other agent value the good so highly? Did I pay too much?

Dutch auctions

Auction	Action protocol	# items
Dutch auction	first-price, open cry, one-shot, **descending**	single

Dutch auction (DA):

- Procedure:
  1. Auctioneer starts with **artificially high value** much above the expected value of any bidder's valuation
  2. Auctioneer continuously lowers the offer price by small value until . . .
  3. Some agent makes a bid for the good equal to the current offer price
  4. The agent has to pay amount of his bid
- DA is also susceptible to **winner's curse**

First-price, sealed-bid auctions

Auction	Action protocol	# items
First-price sealed-bid	first-price, **sealed-bid**, one-shot	single

First-price sealed-bid auction is simplest of all auctions considered here:

- Procedure:
  1. **Single round**, in which bidders **submit their bids privately** to the auctioneer
  2. Auctioneer awards good to agent with **highest bid**
  3. The agent has to **pay amount of his bid**
- **Dominant strategy**: Bid less than its true value
- **Problem**: How much less?
- **No general solution** as it depends on the other agents

Vickrey auctions

Auction	Action protocol	# items
Vickrey auction	**second-price**, sealed-bid, one-shot	single

Vickrey auctions:

- Probably the most counterintuitive auction type
- Procedure:
  1. **Single round**, in which bidders **submit their bids privately** to the auctioneer
  2. Auctioneer awards good to agent with **highest bid**
  3. The agent has to **pay amount of second-highest bid**!
- **Dominant strategy**: Bidders bid their **true valuations**
- **not prone to strategic manipulation**
- **not very popular** in real life, but **very successful** in computational auction systems
- **Problem**: anti-social behavior might occur

Expected revenue

The **expected revenue** of the auctioneer depends on attitudes of auctioneers and bidders:

- **Risk-neutral bidders**: revenue provably identical in all four auctions (under certain simple assumptions)
- **Risk-averse bidders**: **Dutch** and **first-price sealed-bid** auctions best for auctioneer's revenue as risk-averse bidders 'insure' themselves by bidding slightly more than true valuation
- **Risk-averse auctioneers**: Prefer Vickrey or English auction over first-price sealed-bid and Dutch

Important:

- For first result **private values** must exist in agents
- In general, **auction scenario** must carefully be analyzed when choosing auction protocol

Lies and collusion

Ideally:

1. **auctioneer** wants a protocol to be **immune to collusions by bidders**
2. **bidders** want **honesty** to be dominant strategy for auctioneer

Solutions:

1. **immune to collusions** $\Rightarrow$ bidders don't know each other
2. **honest auctioneer** $\Rightarrow$ open-cry auctions or third party handles bids (esp. in case of second price auction)

Further opportunity for auctioneer to manipulate: place bogus bidders, known as **shells** to realize **shill bidding**
$\Rightarrow$ esp. problematic in online auctions such as ebay

Single item auctions overview

Auction	Action protocol	Auctioneer's revenue best when
English auction	first-price, open cry, one-shot, **ascending**	auctioneers risk-averse
Dutch auction	first-price, open cry, one-shot, **descending**	bidders risk-averse
First-price sealed-bid	first-price, **sealed-bid**, one-shot	bidders risk-averse
Vickrey auction	**second-price**, sealed-bid, one-shot	auctioneers risk-averse

Counterspeculation:

- bidders try to gain information either about **true value** of good, or about the **valuations of other bidders**
- If **free** and **accurate**, then every agent would do it
- Otherwise, only if agent's expected result with costly counterspeculation **no worse** than result without

12.3 Combinatorial Auctions

- Bidding languages
- Winner determination
- VCG mechanism I

Combinatorial Auctions

Vickrey auctions work well for single items. How about **resources that are divisible**?

$\Rightarrow$ Combinatorial auctions:

- Generalized model of resource allocation
- Auctioning **bundles of goods** $\mathcal{Z} = \{z_1, \ldots, z_n\}$ (e.g. frequency bands of the mobile phone network)
- New **valuation function** $v_i : \mathbf{2}^{\mathcal{Z}} \to \mathbb{R}$ indicates how much each $Z \subseteq \mathcal{Z}$ is worth to agent i
- Important properties of valuation functions:
  - **Normalization**: $v(\emptyset) = 0$
  - **Free disposal**: $Z_1 \subseteq Z_2 \Rightarrow v(Z_1) \leq v(Z_2)$
- **Outcome**: allocation $Z_1, Z_2, \ldots, Z_n$ of goods being auctioned among the agents

Combinatorial Auctions & social welfare

One natural property combinatorial auctions should satisfy is
⇒ **maximization of social welfare**

$$Z_1^*, \ldots, Z_n^* = \underset{(Z_1,\ldots,Z_n)\in \text{alloc}(\mathcal{Z},Ag)}{\arg\max} sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n)$$

$$\text{where } sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n) = \sum_{i=1}^{n} v_i(Z_i)$$

- **Winner determination**: computing the optimal allocation $Z_1^*, \ldots, Z_n^*$ given the valuations submitted by bidders
- **Strategic manipulation**: agents may not reveal their true valuations (e.g. may overstate the value of bundles)
- **Representational complexity**: exponential in the number of goods (listing all possible valuations of all bundles)
- **Computational complexity**: winner determination is NP-hard even under restrictive assumptions

Bidding languages

As before, most succinct representation schemes for valuation function preferred; first option: **Atomic bid**

- $\beta = (Z, p)$, where $Z \subseteq \mathcal{Z}$ and $p \in \mathbb{R}_+$ is the price
- A bundle of goods Z' **satisfies** (Z, p) if $Z \subseteq Z'$, e.g.:
  - Bundle $\{a, b, c\}$ satisfies the atomic bit $(\{a, b\}, 4)$
  - Bundle $\{b, d\}$ does not satisfy the atomic bid $(\{a, b\}, 4)$
- An atomic bid $\beta = (Z, p)$ defines the valuation function v_β

$$v_\beta(Z') = \begin{cases} p & \text{if } Z' \text{ satisfies } (Z, p) \\ 0 & \text{otherwise} \end{cases}$$

- Not sufficient to express very interesting valuation functions

XOR bids

XOR bids: Specify a number of bids, but par for at most one

- $\beta = (Z_1, p_1)$ XOR ... XOR (Z_k, p_k), for example:
  $\beta_1 = (\{a, b\}, 3)$ XOR $(\{c, d\}, 5)$
  ⇒ "I would pay 3 for a bundle that contains a and b but not c and d; 5 for a bundle with c and d but not a and b; and 5 for a bundle with a, b, c, and d."
- Formally:

$$v_\beta(Z') = \begin{cases} 0 & \text{if } Z' \text{ does not satisfy any of} \\ & (Z_1, p_1), \ldots, (Z_k, p_k) \\ \max\{p_i | Z_i \subseteq Z'\} & \text{otherwise} \end{cases}$$

- XOR bids are fully expressive
- number of bids may be exponential in $|\mathcal{Z}|$
- $v_\beta(Z)$ can be computed in polynomial time

OR bids

OR bids: Combine more than one atomic statement disjunctively

- $\beta = (Z_1, p_1)$ OR ... OR (Z_k, p_k), for example:
  $\beta_1 = (\{a, b\}, 3)$ OR $(\{c, d\}, 5) \Rightarrow v_{\beta_1}(\{a, b, c, d\}) = 8$
- valuation function v for $Z' \subseteq \mathcal{Z}$ is determined w.r.t. atomic bids W so that:
  1. every bid in W is satisfied by Z'
  2. each pair of bids in W has mutually disjoint sets of goods
  3. there is no other subset of bids W' from W satisfying the first two conditions that $\sum_{(Z_i,p_i)\in W'} p_i > \sum_{(Z_j,p_j)\in W} p_j$
- Not fully expressive, consider: $v(\{a\}) = 1, v(\{b\}) = 1, v(\{a, b\}) = 1$
- Can be exponentially more succinct than XOR bids

Winner determination I

Winner determination is **combinatorial optimization problem** $\Rightarrow$ find sets of goods that **maximizes some valuation function**:

- Proven to be **NP-hard** in worst case
- **Optimal** solution calculated using standard technique $\Rightarrow$ **integer linear programming**:
  - **objective function** to maximize: $f(x_1, \ldots, x_k)$
  - subject to **constraints**: $\phi_1(x_1, \ldots, x_k), \phi_2(x_1, \ldots, x_k), \ldots, \phi_l(x_1, \ldots, x_k)$
- With set $\mathcal{Z}$ of goods, set $Ag = \{1, \ldots, n\}$ of agents, and valuation functions $v_1, \ldots, v_n$ (one per agent), $Z \subseteq \mathcal{Z}$:
  - introduce variables $x_{i,Z}$, with $x_{i,Z} = 1$, if bundle Z is allocated to agent i, otherwise $x_{i,Z} = 0$
  - Note: many such variables need to be introduced!

Winner determination II

Winner determination can be encoded as integer linear program:

- maximize: $\sum_{i \in Ag, Z \subseteq \mathcal{Z}} x_{i,Z} v_i(Z)$
- subject to constraints:
  1. $\sum_{i \in Ag, Z \subseteq \mathcal{Z} | z \in Z} x_{i,Z} \leq 1$ for all $z \in \mathcal{Z}$
  2. $\sum_{Z \subseteq \mathcal{Z}} x_{i,Z} \leq 1$ for all $i \in Ag$
  3. $x_{i,Z} \geq 0$ for all $i \in Ag, Z \subseteq \mathcal{Z}$

Meaning of constraints:

1. Don't allocate any good more than once
2. Each agent is allocated no more than one bundle
3. Assures that all variables are either 0 or 1 (together with previous constraints)

This approach works "surprisingly well in many cases." (Wooldridge, p. 307)

The VCG mechanism

Naïve mechanisms are prone to **strategic manipulation**, thus $\Rightarrow$ design mechanism such that, if agents act rationally, dominant strategy is (again) to tell true valuation function

Vickrey-Clarke-Grooves mechanism (VCG mechanism) is generalization of Vickrey's auction from single to divisible goods

Terminology:

- 'Indifferent' valuation function $v^0(Z) = 0$ for all $Z \subseteq \mathcal{Z}$
- $sw_{-i}(Z_1, \ldots, Z_n) = \sum_{j \in Ag: j \neq i} v_j(Z_j)$, social welfare of all agents but i

VCG mechanism II

The Vickrey-Clarke-Grooves mechanism:

1. Agents declare valuation functions $\hat{v}_i$ (may not be true)
2. Mechanism chooses allocation maximizing social welfare:
   $Z_1^*, \ldots, Z_n^* = \arg\max_{(Z_1, \ldots, Z_n) \in \mathrm{alloc}(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n)$
3. Every agent pays to the mechanism or receives from it an amount p_i:
   - compensation' for the utility other agents lose by i participating, or
   - 'reward' for improving the overall utility (then $p_i < 0$)

$$\begin{aligned} p_i = {} & sw_{-i}(Z_1', \ldots, Z_n', \hat{v}_1, \ldots, v_0, \ldots, \hat{v}_n) - \\ & sw_{-i}(Z_1^*, \ldots, Z_n^*, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n), \text{ where} \\ Z_1', \ldots, Z_n' = {} & \arg\max_{(Z_1, \ldots, Z_n) \in \mathrm{alloc}(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, \hat{v}^0, \ldots, \hat{v}_n) \end{aligned}$$

VCG mechanism III

Properties of the VCG mechanism:

- VCG mechanism is **incentive compatible**, i.e. telling the truth is dominant strategy
- For a single goos VCG mechanism **reduces to Vickrey mechanism** $\Rightarrow p_i$ would be the amount of second highest valuation
- Computing VCG payments p_i is NP-hard

VCG mechanism shows that
$\Rightarrow$ **social welfare maximization can be implemented in dominant strategies in combinatorial auctions!**

12.4 Summary

- Thanks

Summary

What we have learned today:

- Different auction types, protocols, and properties thereof
  - English, Dutch, First-price sealed-bid, and Vickrey auction
  - open cry versus sealed-bid, ascending versus descending
  - honesty & collusion
- Combinatorial auctions
  - valuation functions & their properties
  - maximization of social welfare
  - Bidding languages
  - Winner determination
  - The VCG mechanism

Next: Bargaining

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- Dr. Michael Rovatsos, The University of Edinburgh `http://www.inf.ed.ac.uk/teaching/courses/abs/abs-timetable.html`
- Michael Wooldridge: **An Introduction to MultiAgent Systems**, John Wiley & Sons, 2nd edition 2009.