11.1 Motivation

What we've learned so far

Last time we learned about:

▶ Coalition formation
▶ The core of a coalition game
▶ The Shapley value
▶ Different representations for different types of games
  ▶ General coalition games: induced subgraphs & marginal contribution nets
  ▶ Simple games: \((k-)\)-weighted voting games
▶ The Shapley-Shubik power index of simple games

Today:
Coalition Games with Goals & Coalition Structure Formation
So far, utility in coalition games was represented as some numeric value:

\[ \nu : 2^{2^{|\mathcal{G}|}} \rightarrow \mathbb{R} \]

In BDI systems (such as Jason) this is inappropriate. System designers want their agents to achieve some goal(s).

⇒ Qualitative coalition games (QCG)

- Each agent has set of goals and wants one of them to be achieved, but does not care which one
- Agents cooperate to achieve mutually satisfying sets of goals

#### Qualitative coalition games

**Formal model:**
- every coalition C has a set of choices \( V(C) \), i.e. different ways the coalition C could choose to cooperate
- characteristic function of QCG has signature \( V : 2^{2^{|\mathcal{G}|}} \rightarrow 2^{2^{|\mathcal{G}|}} \)

Suppose set of goals \( G' \subseteq G \) is achieved:
- \( G' \) satisfies an agent \( i \) if \( G_i \cap G' \neq \emptyset \), i.e. at least one of its goals is achieved
- \( G' \) is feasible for a coalition \( C \), if \( G' \in V(C) \), i.e. \( G' \) is one of the choices available to \( C \)
- Coalition \( C \) is successful, if \( C \) can cooperate in such a way that \( G' \) satisfies every member of \( C \)

**Propositional logic representation** is complete, but not guaranteed to be succinct.

#### Coalition resource game

QCGs say nothing about where the characteristic function comes from, or how it is derived for a given scenario.

⇒ The coalition resource game framework (Wooldridge & Dunne, 2006):

- Simple idea: To achieve a goal requires consumption of resources and each agent is endowed with a profile of resources
- Coalitions form to pool resources and achieve mutually satisfactory set of goals

Interesting questions:
- Theoretical: Can a pair of coalitions achieve their goals whilst staying within their respective resource bounds?
- Real world: Can some countries achieve their economic objectives without consuming too many pollution-producing resources?
11.3 Coalition Structure Formation

So far, every agent acts strategically just as in non-cooperative games, attempting maximization of own utility.

⇒ What if one designer owns all agents?

▶ Performance of single agents perhaps not as important
▶ Better maximize social welfare of the system
▶ Maximizing social welfare ⇒ maximizing the sum of the values of individual coalitions

A coalition structure is a partition of the overall set of agents $Ag$ into mutually disjoint coalitions.

Example, with $Ag = \{1, 2, 3\}$:

- Seven possible coalitions:
  \[
  \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}
  \]

- Five possible coalition structures:
  \[
  \{\{1\}, \{2\}, \{3\}\}, \{\{1\}, \{2, 3\}\}, \{\{2\}, \{1, 3\}\},
  \{\{3\}, \{1, 2\}\}, \{\{1, 2, 3\}\}
  \]

Given game $G = (Ag, \nu)$, the socially optimal coalition structure $CS^*$ is defined as:

\[
CS^* = \arg \max_{CS \in \text{partitions of } Ag} V(CS)
\]

where

\[
V(CS) = \sum_{C \in CS} \nu(C)
\]

Unfortunately, there are exponentially more coalition structures over the sets of agents $Ag$ then there will be coalitions over $Ag$

⇒ Exhaustive search is infeasible (in the worst case!)

Sandholm et al. (1999) developed a technique that guarantees to find a coalition structure that is within some provable bound from the optimal one.
11.4 Summary

- Thanks

Summary

What we have learned today:
- Coalition Games with Goals
  - Goals, not numeric utilities, as targets for agents
  - Qualitative coalition games
  - Coalition resource game
- Coalition Structure Formation
  - Maximizing social welfare, instead of individual agent’s utility
  - Number of coalition structures exponential in the number of coalitions

Next (on Wednesday): Allocating Scarce Resources

Acknowledgments

These lecture slides are based on the following resources: